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SOME ALGEBRAIC PROPERTIES OF THE FUZZY FUNCTIONS

T. T. BUHĂESCU*

1. **Introduction.** The notion of fuzzy set introduced by L. A. Zadeh in 1965, led to a widely used theory in various humanistic and technical sciences. Modelling in a fuzzy sense requires, among other conditions, the adequate extension of some classical mathematical concepts, or, in other words, their „fuzzification". Thus in the literature of the field, the *„fuzzification"*. of the function notion appears in the following main senses : common functions with fuzzy properties or satisfying fuzzy restictrions ; fuzzy extensions of the commom functions ; fuzzy functions with nonfuzzy variables ; common function of fuzzy variables (see *D.* Dubois and H. Prade [3]). The last two types are known as fuzzy functions (see S. S. L. Chang [2]) and fuzzy functions (see M. A. Erceg [4]). Fuzzy functions reprezent the normal generalization of multifunctions.

In this paper it is our intention to deal with the algebraic properties of the fuzzy functions on analogy with the multifunctions theory.

2. **Preliminaries.** The notion of fuzzy set, as well as the operations of reunion, intersection and complementation are considered in the sense Zadeh [6] uses them. We note the interval $[0,1]=L$ and max and min operators with $\vee$ and $\wedge$, respectively. The fuzzy sets class in X is isomorphic with $L^X=\{f|f: X\to L\}$, thus a fuzzy set identifying itself with its memberschip function, both being noted with the same letter.

Definition 2. 1. Let X and Y be arbitrary sets. The function $F: X\to L^Y$ is called fuzzy function. The function $F^{-1}: Y\to L^X$ so that $F^{-1}(y)(x)=F(x)(y)$ for all $x\in Y$ is called the inverse function of F.

Definition 2.2. If $A\subseteq$ P, $B\subseteq Y$ and $F: X\to L^Y$ then :

(1) $$F(A)=\bigcup_{x\in A}F(x)\ ;\ F^{-1}(B)=\bigcup_{y\in B}F^{-1}(y)\ ;\ F(\Phi)=\Phi\ ;\ F^{-1}(\Phi)=\Phi$$

Definition 2.3 If A and B are fuzzy sets in X, the cartesian product AXB is a fuzzy set in X^2 given under this form :

(2) $$(AXB)\ (x_1, x_2)=\min\ (A(x_1), B(x_2))$$

Definition 2.4. Let be $F_i: X\to L^Y$, $i=1, 2$ two fuzzy functions. The reunion, intersection and the cartesian product $F_1\cup F_2:\ X\to L^Y$; $F_1\cap F_2: X\to L^Y$ and $F_1\times F_2: X\to L^{Y^2}$ are given punctually by: $(F_1\cup F_2)\ (x)=F_1(x)\cup F_2(x)$; $(F_1\cap F_2)\ (x)=F_1(x)\cap F_2(x)$; $(F_1\times F_2)\ (x)=F_1(x)\times F_2(x)$.

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(20) $$F^-(B_1) \cup F^-(B_2) = F^-(B_1 \cup B_2)\ ;\ F^-(B_1) \cap F^-(B_2) \subseteq F^-(B_1 \cap B_2)$$

Proof

a) If $x \in {}^cF^+(B_1)$ then $x \notin F^+(B_1)$ therefore $F(x) \not\subset B_1$. There is at least one $y_0 \in Y$ so that $F(x)(y_0) > B_1(y_0)$. If $B_1(y_0) = 0$ then $F(x)(y_0) > 0$ consequently we have the relation:

(21) $$F(x)(y_0) \Lambda (1 - F_1(y_0)) \neq 0$$

If $B_1(y_0) > 0$ cannot equal 1, because $F(x)(y_0) > 1$ is not possible. This shows that $1 - B_1(y_0) > 0$ therefore (21) is true, that is $F(x) \cap {}^cB_1 \neq \Phi$ consequently $x \in {}^cF^-(B_1)$, and the first inclusion in (18) is true. Let there be $x \in {}^cF^-(B_1)$. This implies $x \notin F^-(B_1)$, therefore $F(x) \cap B_1 = \Phi$, a relation which is equivalent with $F(x)(y) \Lambda B_1(y) = 0$ for all $y \in Y$. As $F(x)$ and B_1 are nonvoid fuzzy sets, it follows that:

(22) $$(\text{supp } F(x)) \cap (\text{supp } B_1) = \Phi$$

(for $A \in L^X$, supp $A = \{x \mid x \in X,\ A(x) \neq 0\}$)

From (22) we can conclude that supp $F(x) \leqslant {}^c(\text{supp } B_1)$

$$\text{As } ({}^cB_1)(y) = \begin{cases} 1 - B_1(y) & \text{if } y \in \text{supp } B_1 \\ 1 & \text{if } {}^cy \in {}^c(\text{supp} B_1) \end{cases}$$

from the above inclusion we can obtain $F(x) \subseteq B_1$ which shows that $x \in F^+({}^cB_1)$ thus the second inclusion in (18), being proved.

b) If $x \in F^+(B_1) \cup F^+(B_2)$ then $F(x) \subseteq B_1$ or $F(x) \subseteq B_2$ which leads to $F(x) \subseteq B_1 \cup B_2$ consequently $x \in F^+(B_1 \cup B_2)$ the inclusion in (19) being proved.

c) If $x \in F^-(B_1) \cap F^-(B_2)$ then $F(x) \cap B_1 \neq \Phi$ or $F(x) \cap B_2 \neq \Phi$ which leads to $F(x) \cap (B_1 \cup B_2) \neq \Phi$ consequently $x \in F^-(B_1 \cup B_2)$. Let be $x \in F^-(B_1 \cup B_2)$ that is $F(x) \cap (B_1 \cup B_2) \neq \Phi$. Using the distributivity law, we obtain $(F(x) \cap (B_1) \cup (F(x) \cap B_2) \neq \Phi$ and we have $F(x) \cap B_1 \neq \Phi$ or $F(x) \cap B_2 \neq \Phi$. From the last one, it fallows that $x \in F^-(B_1)$ or $x \in F^-(B_2)$ therefore $x \in F^-(B_1) \cup F^-(B_2)$. Similarly, we have inclusion in (20) and equality in (19).

Theorem 3.8. Let $F: X \to L^Y$ be a fuzzy function, so that $F(x) \neq \Phi$ for all $x \in X$. The collection of fuzzy sets in Y having the property: $F^+(P) = F^-(P)$ called pure fuzzy sets related to F and noted $\mathcal{D}_F$, is a latice with the reunion and intersection operations having the first and the last element and, furthemore it satisfies the relations:

(23) $${}^cF^-(B) = {}^cF^+(B) \subseteq F^+(B) \subseteq F^-({}^cB),\ \forall B \in \mathcal{D}_F$$

Proof Let $\mathcal{D}_F = \{P \mid P \in L^Y,\ F^+(P) = F^-(P)\}$ be. As $F(x) \neq \Phi$ for all $x \in X$; it fallows that Y and Φ are in $\mathcal{D}_F$. Let there be now $P_1, P_2 \in \mathcal{D}_F$. From definition 2.6 and theorem 3.7 we have:

(24) $$F^+(P_1 \cup P_2) \subseteq F^-(P_1 \cup P_2)\ ;\ F^+(P_1) \cup F^+P_2) \subseteq F^+P_1 \cup P_2)$$

On the other hand, from the hypothesis and 3.7 we obtain:

(25) $$F^+(P_1) \cup F^+(P_2) = F^-(P_1) \cup F^-(P_2) = F^-(P_1 \cup P_2) \subseteq F^+(P_1 \cup P_2)$$

The first inclusion in (24) and the last in (25) lead to:

(26) $$F^{+}(P_1 \cup P_2) = F^{-}(P_1 \cup P_2)$$

therefore $P_1 \cup P_2 \in \mathcal{D}_F$.

If $x \in F^{-}(P_1 \cap P_2)$ then $F(x) \cap (P_1 \cap P_2) \neq \Phi$ wich leads to $(F(x) \cap P_1) \cap \cap (F(x) \cap P_2) \neq \Phi$ consequently $F(x) \cap P_1 \neq \Phi$ and $F(x) \cap P_2 \neq \Phi$ that is $x \in F^{-}(P_1)$ and $x \in F^{-}(P_2)$. Taking into account the hypothesis, we have $x \in F^{+}(P_1)$ and $x \in F^{+}(P_2)$, which leads to $x \in F^{+}(P_1) \cap F^{+}(P_2)$. From theorem 3.7 $F^{+}(P_1) \cap \cap F^{+}(P_2) = F^{+}(P_1 \cap P_2)$, consequently $x \in F^{+}(P_1 \cap P_2)$, appartenance leading to the inclusion:

(27) $$F^{-}(P_1 \cap P_2) \subseteq F^{+}(P_2 \cap P_2)$$

The inverse inclusion results from 2.6, thus giving:

(28) $$F^{+}(P_1 \cap P_2) = F^{-}(P_1 \cap P_2)$$

therefore $P_1 \cap P_2 \in \mathcal{D}_F$

From the hypothesis, it follows that ${}^cF^{+}(P) = {}^cF^{-}(P)$ for all $P \in \mathcal{D}_F$ which together with the inclusions (18), led to (23).

Theorem 3.9. Let be $F: X \to L^Y$ a fuzzy function with $F(x) \neq \Phi$ for all $x \in X$. The common subsets collection of X checking the relation $F^{-}(F(S)) = S$ is a latice relative to reunion and intersection with the first and last element. This latice is called the latice of stable subsets related to the fuzzy function F.

Proof. Let there be $\mathcal{S}_F = \{S \mid S \subseteq X, F^{-}(F(S)) = S$. From 2.6, we see that for all $S \subseteq X$ there is $F^{-}(F(S)) \supseteq S$. The hypothesis that $F(x) \neq \Phi$ for all $x \in X$ leads to $\Phi, X \in \mathcal{S}_F$.

Let there be $S_1, S_2 \in \mathcal{S}_F$. From 3.1., we have $F(S_1 \cup S_2) = F(S_1) \cup F(S_2)$ and taking into account 3.7, we obtain:

(29) $$F^{-}(F(S_1 \cup S_2)) = F^{-}(F(S_1) \cup F(S_2)) = F^{-}(F(S_1)) \cup F^{-}(F(S_2)) = S_1 \cup S_2.$$

therefore $S_1 \cup S_2 \in \mathcal{S}_F$.

If $A \subseteq B$ then $F^{-}(A) \subseteq F^{-}(B)$. Appling this inclusion, 3.2 and 3.7 it fallows that:

(30) $$F^{-}(F(S_1 \cap S_2)) \subseteq F^{-}(F(S_1) \cap F(S_2)) \subseteq F^{-}(F(S_1) \cap F^{-}(F(S_2)) = S_1 \cap S_2.$$

We have mentioned above the reverse inclusion, therefore $S_1 \cap S_2 \in S_F$, which ends the proof.

Remark 3.10. It is natural to compare the fuzzy function properties with some corresponding ones of the multifunctions. Thus 3.1. and 3.2. are the same of multifunctions. The properties included in 3.3., 3.4., 3.6. are extensions of the same properties of the multifunctions based on fuzzy functions compound and on the notions of q-univocity and k-injeticvity. There are differences in the properties related to complementary which can be seen in 3.7., 3.8 and 3.9.

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UNELE PROPRIETĂŢI ALGEBRICE ALE FUNCŢIILOR FUZZY

(Rezumat)

Noţiunea de funcţie fuzzy generalizează în mod natural pe aceea de multifuncţie.
Multifuncţiile semiunivoce şi injective se generalizează prin funcţiile fuzzy q-univoce, respectiv k-injective.
Sînt relevate proprietăţi algebrice ale funcţiilor fuzzy în analogie cu proprietăţile multifuncţiilor. Unele proprietăţi sînt identice. altele sînt specifice funcţiilor fuzzy.

QUELQUES PROPRIÉTÉS ALGÉBRIQUES DES FONCTIONS FUZZY

(Résumé)

La notion de la fonction fuzzy est une généralisation de la fonction multivoque.
Les fonctions multivoques semiunivoques et injectives sont générolisées par les fonctions fuzzy q-univoques et k-injectives. Les propriétés algébriques des fonctions multivoques sont toujours vraies pour les fonctions fuzzy. Il y a des propriétés des fonctions fuzzy qui ne sont pas vraies pour les fonctions multivoques.

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CONTRIBUTI ALLA TEORIA DEI NUMERI ED APPLICAZIONI. Nota I. CARATTERIZZAZIONE DEI NUMERI NATURALI COMPOSTI IN RELAZIONE AI NUMERI PRIMI

DI

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Dedichiamo questa Memoria all'infinitamente compianto MAURO PICONE, nel 95° anniversario dalla sua nascità.

1. **Introduzione.** E'ben noto che la gamma di svariati contributi alla Teoria dei numeri ad alle sue applicazioni cresce rapidamente. Basta citare anzitutto : 1. L'esistenza delle riviste specializzate in dominio, tali, ad es., *Discrete mathematics* (North Holland Publ. Co., Amsterdam, vol. I : 1971). *Fibonacci quarterly* (Fibonacci Assn, St. Mary's Coll., CA 94575 ; questo periodico inserisce lavori concernenti numeri interi con proprietà speciali. Vol. I : 1963) ; *Journal of combinatorial theory e journal of number theory* (Academic Press, New York and London, vol. I : 1968 e, rispettivamente, 1968—1969) ; 2°. La spartizione col tempo della Teoria dei Numeri nella Teoria additiva dei numeri primi, Teoria geometrica dei numeri, Teoria analitica dei numeri ed altri ancora ; 3°. L'attenzione accordata all'approfondimento dell'ottavo problema di Hilbert concernente la distribuzione dei numeri primi e l'ipotesi di Riemann, [1], [2] ; 4°. L'organizzazione abbastanza regolare dei congressi, convegni, conferenze ecc., tali, ad es., *„The Sixth Australian Conference on Combinatorial Mathematics"*, Armindale, Australia (A. F. Horadam e W. D. Wallis, editori, Aug. 1978 ; Proc., Springer-Verlag, 1979, pp. IX+206) e *„The Southern Illinois Number Theory Conference"* (Melvin B. Nathanson, editor, Marzo 30 e 31, 1979 ; Proc., Springer-Verlag, 1979, pp. V+342) e ciò senza parlarne troppo del *„hobby"* concernente la Teoria dei numeri di quasi tutti i matematici, almeno per un certo periodo di tempo (Vedansi, ad es. [3] — [8]).

In ciò che riguarda la gamma di applicazioni, ci limiteremo costi alla riproduzione del seguente teorema di Mangeron concernente le equazioni polivibranti da lui introdotti un mezzo secolo fa, [9] : Il numero N (λ) degli autovalori, inferiori ad una quantità prefissata λ, dell'equazione polivibrante

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$$(1) \qquad Lu - \lambda u = 0, \quad L \equiv \partial^4 / \partial x^2 \partial y^2 \equiv M_2^{(2)},$$

per il dominio quadrato di superficie a e per la condizione $u=0$ sulla frontiera di esso è asintoticamente uguale a

$$\frac{a^2}{2\pi^2} \sqrt{\lambda} \log \lambda$$

ed alla menzione del rapporto tecnico non pubblicato, [10], che si referisce alla programmazione tramite le calcolattrici elettroniche del calcolo degli interi successivi spettanti ai gruppi di meccanismi piani costituiti dai numeri vieppiù crescenti di articolazioni.

II. **Cenni ed addenda a ciò che si conosce dei numeri naturali composti in relazione ai numeri primi componenti.** Dato un numero intero $n \in N_0$ è ben noto che riconoscere se n sia primo o composto non è cosa semplice, per n elevato, guadagnandosi tale verità attraverso la ripetizione di una classica indagine, più o meno modificata nel corso dei secoli, la quale, per numeri piuttosto alti, diviene pressochè *impraticabile*. E'di questi ultimi tempi la notizia che presso l'Università di California è stato rinvenuto il più alto primo fra quelli conosciuti, ottenuto a prezzo di laboriosi e complessi calcoli, eseguiti anche con elaboratori eletronici[6]. Il problema che si occupa poi della distribuzione dei numeri primi in certi intervalli numerici, affrontato già da tempo con i meszi più riposti dell'Analisi Matematica e del Calcolo della Probabilità, non può essere molto semplificato, e risolto elementarmente, se non con indagini dirette. Non valgono perciô i caratteri di divisibilità, transformati e adattati, e neppure i *crivelli* usati per ottenere i noti *atlanti* sui numeri primi, [11]. Essi bastanoper un numero molto limitato di cifre, si resta impotenti nel riconoscimento, se n contiene milioni, o miliardi, o miliardi di miliardi di cifre, se esso sia primo o composto, anche con l'impiego dei calcolatori più sofisticati[6].

Molti tentativi sono stati effettuati dagli studiosi della Teoria dei Numeri nella ricerca di metodi agili per dimostrare :

a) L'esistenza di infiniti numeri primi nell'insieme numerico dei valori di una determinata formula algebrica, o funzione aritmetica intera, $y=f(n)$, al variare del numero intero naturale n ;

b) attraverso proprietà generali, dei particolari *criteri sufficienti* sui numeri primi, per evitare di aplicare la divisibilità ad un dato numero n per altri numeri primi precedenti p, fino al massimo $\overline{p}$ fra essi, il cui quadrato superi n $(\overline{p} > n)$, [12].

Le due ricerche si integrano a vicenda. Richiamiamo qualcuno di questi tentativi : a') Alcune formule construttive possono servire talvolta

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[6]) Il nuovo numero puo essere indicato con $2^{21701} - 1$ (esso è un numero di Mersenne), il quale risulta ben più elevato del numero *record* precedente, equivalente a $2^{19739} - 1$. Esso è stato ottenuto da due giovani ricercatori il 30 ottobre 1978, dopo tre anni di calcoli.

[4]) I numeri primi deducibili da quella formula sono : 3, 5, 17, 257, 65537 ; essi furono utilizzati da C. F. Gauss nella sua teoria ciclometrica per l'iscrizione dei poligoni regolari in una circonferenza con l'uso della riga e del compasso. Può darsi che esistono altri al di sopra din $n > 5$. Per $n = 5$, si deduce che

$$2^{32} + 1 = 641 . 6700417 .$$

a mettere in luce la generazione, al variare di un parametro intero di insiemi limitati o illimitati di numeri primi. Cosi i *numeri di Fermat* $y=2^{2^n}+1$ risultano primi per $n=0, 1, 2, 3, 4$, mentre Euler nel 1713 scoperse che per $n=5$, y è composto, divisibile per 641, ; b') Due altre espressioni aritmetiche curiose forniscono numeri primi al variare del numero intero positivo n,

$$y(n)=n^2-n+41\,; \qquad z(n)=n^2-79n+1601,$$

nella prima facendo $n=1, 2, \ldots, 40$, nella seconda $n=1, 2, \ldots, 79$, ma che, per n maggiore dei valori indicati, danno numeri composti. Naturalmente di queste espressioni funzionali se ne potrebbero ottenere altre, ma non si è sicuri che esse presentino infiniti valori aritmetici coincidenti con numeri primi.

Nei testi di Teoria dei Numeri appaiono proposizioni concernenti i numeri interi della forma $N=ak+b$ (a, b coefficienti interi naturali, k parametro intero positivo, ad esempio), per *i* quali si debbono precisare : 1°) *i* valori di k per cui essi risultino primi e dimostrare che : 2°) — il loro numero sia infinito (teorema di Dirichlet, il fondatore della „*Teoria analitica dei numeri*"). In particolare sarebbe utile dimostrare la proposizione : „Esistono infiniti numeri primi del tipo $y(n)=2^n \cdot m+a$, con n intero positivo, m parametro variabile anche esso intero positivo, e $a \gtrless 0$, intero minore di 2^n e primo con m'', il che è possibile. Rammentiamo ancora, per inciso, *i* numeri primi di quella forma per $m=1$ e $a=-1$; *i* quali sono denominati (l'abbiamo già detto) *numeri di Mersenne* ($n \in N$, primo), [13]. Il matematico P. Mac Carthy, [14], riferì nel 1957 di aver classificato in questo gruppo i numeri primi corrispondenti a $n=2, 3, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281$, e ancora $n=4423$. Ma il gruppo si è notevolmente allargato oggi con altri numeri primi, tra i quali annoveriano le punte estreme per $n=19937$ e $n=21701$[5].

Interessanti sono i seguenti criteri riguarfdanti questi particolari numeri : 1') Se m è un numero primo dispari, i divisori di 2^m-1 (ove questo non sia primo) sono della forma $2mx+1$; x essendo un conveniente intero positivo e 2') Se m è un numero primo e $2m+1$ è un numero primo della forma $8h+7$($h>0$, parametro intero variabile), allora 2^m-1 è composto ed è divisibile per 2^m+1. I numeri primi della forma 2^k-1 intervengono nella ricerca dei numeri cosidetti numeri *perfetti*. Mentre la dimostrazione dei criteri 1') è dovuta a L. Euler, un'altra proprietà interessante su questi speciali numeri si deduce da un teorema di E. Lucas che rinunciamo di riprodurre qui.

III. **Alcuni criteri sufficienti per la qualifica di numero primo** (o semplice). Risaliamo al secolo scorso allorchè vennero approfondite alcune ricerche „*sui grandi numeri primi*", dovuti sopratutto a E. Lucas, [15]. Questi parte dal fatto che : „se il gss (p, a)$=p-1$, allora p è primo" (p intero positivo, a intero positivo $\neq 1$, *aep* assegnati entrambi, a primo con p. Si dice allora cheaè *radice primitiva* di p. Fra *i* numeri a il più semplice è 2.

Ora, puessere 2 radice primitiva di p dispari ? E in quale circostanza ? Il Lucas risponde con alcuni criteri a tale domanda.

1° *criterio*: „Se p è un numero primo dispari, allora $2p+1$ risulta primo se è valida la congruenza $2^{2p} \equiv 1 \pmod{2p+1}$“. Il critierio or ora enunciato si può scindere in altri due criteri subordinati che diano della proposizioni equivalenti al teorema di Wilson sui numeri primi. Precisamente, il numero p potrebbe essere della forma a) $p=4m+1$, *b*) $p=4m+3$. Sia $p=4m+1$, allora $2p+1=8m+3$ e dunque $2^{nm+1} \equiv -1 \pmod{8m+3}$ e $8m+3$ risulterà primo ; in vece per $p=4m+3$ risulta $2p+1=8m+7$ e, allora, se $2^{4m+3} \equiv 1 \pmod{8m+7}$ porta ad affernare che $8m+7$ è primo, in quantpo il gss $(2p+1\,;\,2)=4m+3$ e l'espressione $[4m+3\,:]^{2}1 \pmod{2p+1}$ è perfettamente equivalente al teorema di Wilson. Perciò il primo criterio di Lucas si puô scindere negli altri due:

a$_1$) Se $2^p \equiv -1 \pmod{2p+1}$, allora $2p+1$ è primo, se $p=4m+1$ è primo;
a$_2$) Se $2^p \equiv 1 \pmod{2p+1}$, allora $2p+1$ è primo, se $p=4m+3$ è primo [16]

2° *criterio*: „Se p è un numero prino, naturalmente dispari, perchè $4p+1$ sia primo basta che sia $2^2p+1 \equiv 0 \pmod{4p+1}$, il che equivale ad affermare la medesima cosa per la congruenza:

$$2^p+1 \equiv \pm 2^{(p-1)/2} \pmod{(4p+1)}“.$$

3° *criterio*: „Se p è un numero primo dispari, perchè $6p+1$ sia primo, è sufficiente che si abbia:

$$2^{3p}-1 \equiv 0 \pmod{(6p+1)}, \text{ se } p \text{ è uguale a } 4m+1\,;$$

$$2^{3p}+1\ \ 0 \equiv \pmod{(6p+1)}, \text{ se } p \text{ è uguale a } 4m+3“.$$

4° *criterio*: „Se p è un numero primo dispari, perchè $10p+1$ sia primo è sufficiente che si abbia:

$$2^{5p}+1 \equiv (\mathrm{mod}\ (10p+1), \text{ se } p=4m+6\,;$$

$$2^{5p}-1 \equiv \pmod{(10p+1)}, \text{ se } p=4m+3.$$

Gli autori, chiudendo questa Nota, sono ben consci della difficolte di far puntare se put in grandi linee almeno alcuni lati dell'enorme moli di lavori compiuti in tre secoli a questa parte nel dominio della Teoria de Numeri. Pur sottolineando il fatto che nel testo si sono puntati alcuni nuovi commenti, parallelamente ai svariati quesiti, il cui numero accrescerà notevolmente nelle note successive ove saramo esposti contributi concernenti parti-

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[8]) Il gss$(p, a)=x$ (si legge: il gaussiano di p alla base a) è il *minimo* numero naturale per cui $a^x \equiv 1$ (modp). Nel caso che $x=p-1$, ciò vuol dire che il gruppo $G(a, a^2, \ldots, a^{p-1})$ coincide, senza tener conto dell'ordine, con $G(1, 2, \ldots, p-1)$. Per $a=2$, il gruppo diviene $(2, 2^2, \ldots, 2^{p-1})$, e allora è evidente che deve essere $2^{\frac{p-1}{2}} \equiv -1 \pmod{p}$.

[9]) Nel caso a_1) si ponga mente all'esempio $4.13+1=53$ e a $8.13+3=107$, nel caso a_2) a $4.2+3=11$ e a $8.2+{}^7=23$. Ora, per i criteri di Lucas si puo dire che, se $2p+1$ è primo, è chiaro che $2^{2p} \equiv 1 \pmod{2p+1}$, ma quest'ultima congruenza, che è *necessaria* per i numeri primi (teorema di Fermat), *non è* poi *sufficiente*, giacchè asistono anche dei numeri naturali composti m soddisfacenti alla congruenza $a^{m-1} \equiv 1 \pmod{m}$ per determinati valori assunti da a, intero positivo, primo con m, [16], [17].

colari congruenze binomiali ed analogie con i numeri naturali composti, seqguiti dai commenti sul teorema di Jacobi, dalle tabelline numeriche concernenti i caratteri di dicisibilità per i numeri primi ed altri ancora. Ci promettiamo tuttavia di continuar con alcuni quesiti, poichè i lavori spettanti alla Teoria dei Numeri e specialmente alla teoria analitica dei numeri, ai problemi concernenti i numeri primi, espongono generalmente ciò che l'autore è capace di dimostrare, allorchè raramente si fa menzione ai fini più ambiziosi di nuove congetture, di nuovi problemi atti ad in citare la genesi delle nuove idee e dunque all'andar del tempo di arrichire la Scienza con nuovi risultati.

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CONTRIBUŢII LA TEORIA NUMERELOR ŞI APLICAŢII, I. CARACTERIZAREA NUMERELOR NATURALE COMPUSE ÎN RELAŢIE CU NUMERE PRIME

(Rezumat)

Autorii, punctînd stadiul actual al unei serii de probleme de teoria numerelor şi legîndu-l de achiziţiile de seamă în domeniu realizate pe parcursul ultimelor trei secole, se mărginesc la a face unele observaţii menite să contribuie la intensificarea cercetărilor privitoare la Teoria numerelor şi aplicaţiile acesteia.

CONTRIBUTIONS TO NUMBER THEORY. I. CHARACTERIZATION OF COMPOSED NATURAL NUMBERS IN THEIR RELATIONSHIP WITH PRIME NUMBERS

(Summary)

The authors are poiting out a series of new and old results concerning Number Theory and are listing some questionmarks, just to obtain a large picture of the suspected truth, against which one can test ideas.

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ФУНДАМЕНТАЛЬНЫЕ РЕШЕНИЯ ЗАДАЧИ КОШИ ДЛЯ ИНВАРИАНТНЫХ ПАРАБОЛИЧЕСКИХ ОПЕРАТОРОВ НА РИМАНОВЫХ МНОГООБРАЗИЯХ

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Изучение смешанных задач для параболических операторов в областях с негладкой границей приводит к необходимости простроения фундаментальных решений задачи Коши в специальных римановых пространстаах (на специальных риманоаых многообразиях) $R_n^{(m)} = R_2^{(m)} \times E_{n-2}$ $(2 \leqslant m \leqslant \infty)$, где $R_2^{(m)}$ — римановая поверхность функции $\bar{\bar{w}} = \sqrt[m]{\bar{z}}\,(z = =\bar{x}_1 + i\bar{x}_2)$, $R_2^{(\infty)}$— римановая поверхность функции

$$\bar{\bar{w}} = \ln \bar{\bar{z}} = \lim_{m \to \infty} m(\sqrt[m]{\bar{\bar{z}}} - 1),$$

а $E_{\bar{k}}$ — $\bar{\bar{k}}$ — мерное эвклидовое пространство. Основным орудием построения таких есть интегральное представление обобщенной функции (δ-функции).

В эвклидовом пространстве E_n обозначим через

$R^2 = \sum_{i=1}^{n} (\bar{x}_i - \xi_i)^2 \equiv |\bar{x} - \xi|^2$ — квадрат расстояния между точками $\bar{x}$ и ξ,

$w_n = 2(\pi)^{n/2}\Gamma^{-1}\left(\dfrac{n}{2}\right)$— величину площади единичной сферы, $j_\nu(\bar{s})$ — нормированную функцию Бесселя I-го рода действительного аргумента порядка ν

$$j_\nu(\bar{s}) = 2^\nu \Gamma(\nu + 1)\bar{s}^{-\nu} I_\nu(\bar{s}),\ j_\nu(0) = 1,\ j_\nu^1(0) = 0,$$

$I_\nu(\bar{s})$ — обычная функция Бесселя I-го рода [1].

Теорема 1. Если при любом $\varepsilon > 0$ для непрерывной по совокупности переменных функции $f(t, \lambda)$, для которой $f(0, \lambda) = 1$, интеграл

$$\frac{w_n}{(2\pi)^n} \int_0^\infty f(t, \lambda)\lambda^{n-1} j_{\frac{n-2}{2}}(\lambda R)\,d\lambda$$

рааномерно сходится при $t > \varepsilon$ к обычной функции $G(t, \bar{x}, \xi)$, то в смысле теории обобщенных функций

$$\lim_{t \to +0} G(t, \bar{\bar{x}}, \xi) = \delta_\xi \equiv \delta(\bar{\bar{x}} - \xi). \qquad \text{(I)}$$

Доказательство. Пусть f — инвариантная относительно вращений вокруг начала координат функция, для которой существует опреобразование фурье

$$\tilde{f}(x-\xi)=\frac{1}{(2\pi)^n}\int_{E_n} f(|w|)e^{i(\xi-\bar{\bar{x}},\, w)}\,dw \tag{2}$$

и ρ — элемент ортогональной группы $O(n)$. Тогда $(\xi-\bar{x}, w)=(\rho^{-1}(\xi-\bar{\bar{x}}), \rho^{-1}w)$ и

$$\tilde{f}(\rho(x-\xi))=\frac{1}{(2\pi)^n}\int_{E_4} f(|w|)e^{i(\xi-\bar{\bar{x}},\, \rho^{-1}w)}\,dw \tag{3}$$

Сделав в интеграле (3) замену переменных по формуле $w=\rho\bar{w}$, получим, учитывая инвариантность функции $f(|w|)$, что

$$\tilde{f}(\rho(\bar{\bar{x}}-\xi))=\frac{1}{(2\pi)^n}\int_{E_n} f(|\bar{\bar{w}}|)e^{i(\xi-x,\, \bar{w})}\,d\bar{\bar{w}}=\tilde{f}(\bar{\bar{x}}-\xi),$$

т.е. функция $\tilde{f}$ — инвариантная относительно вращений, а значит $\tilde{f}$ является функцией эвклидового расстояния $R: \tilde{f}(x-\xi)=f(\tilde{R})$. Полагая в (2) $\bar{\bar{x}}_1-\xi_1=\bar{\bar{\jmath}}$, $\bar{\bar{x}}_2=\xi_2, \ldots, \bar{\bar{x}}_n=\xi_n$ и переходя к сферическим координатам, имеем [1] :

$$\tilde{f}(\ )=\frac{1}{(2\pi)^n}\int_{E_n} f(|w|)e^{i\bar{\bar{p}}w_1}\,dw=\frac{1}{(2^m)^n}\,\frac{2\pi^{\frac{n-1}{2}}}{\Gamma\frac{(n-1)}{2}}\int_0^\infty f(\lambda)\lambda^{n-1}\times$$

$$\times\int_0^{\pi} e^{i\rho\lambda\cos\varphi_1}\sin^{n-2}\varphi_1 d\varphi_1 d\lambda=\frac{\omega_n}{(2\pi)^n}\int_0^\infty f(\lambda)\lambda^{n-1}j_{\frac{n-2}{2}}(\lambda R)d\lambda.$$

Следовательно, при $t>\varepsilon$

$$\frac{w_n}{(2\pi)^n}\int_0^\infty f(t,\lambda)\lambda^{n-1}j_{\frac{n-2}{2}}(\lambda R)d\lambda=\frac{1}{(2\pi)^n}\int_0 f(t,|w|)e^{i(\xi-\bar{\bar{x}},\, w)}\,dw,$$

причем из сходимости одного интеграла вытекает сходимость другого. Теперь формула (I) вытекает из того факта, что преобразование фурье функции δ_ξ равно $e^{i(\xi, w)}$ [2].

Рассмотрим теперь случай риманового многообразия $R_n^{(m)}$. В $R_2^{(m)}$ будем пользоваться полярными координатам $(r, \varphi$. При этом будем говорить, что точка с полярными координатами $(0\leqslant r<\infty, \varphi)$ лежит на первом листе $R_2^{(1)}$ римановой поверхности, если $|\varphi|\leqslant\pi$ и, вообще, на $(\bar{k}+1)$ — ом листе $R_2^{(\bar{k}+1)}$ $(\bar{\bar{k}}=1, 2, \ldots, m-1)$, если

$$(2\bar{k}-1)\pi<\varphi<(2\bar{\bar{k}}+1)\pi. \tag{4}$$

Тем самым положение произвольной точки в $R_n^{(m)}$ определяется n — униформизующими параметрами $(r, \varphi, \bar{\bar{x}}_3, \ldots, x_n)$, а координаты фиксированнойточки$(\xi_1, \xi_2, \ldots, \xi_n)$ будут обозначаться через $(\rho, \varphi_0, \xi_3, \ldots, \xi_n)$. Точку, лежащую на к — ом экземпляре $R_m^{(\bar{\bar{k}})} = R_2^{(\bar{\bar{k}})} \times E_{n-2}$ риманового многообразия $R_n^{(m)}$ обозначим через x^k. Таким образом, $x^{\bar{\bar{k}}} \in R_n^{(\bar{\bar{k}})}$, причем первый экзамляр риманового многообразия $R_n^{(m)}$ мы будем отождествлять с эвклидовым пространством E_n..

Определение I. Будем говорить, что на многообразии $R_n^{(m)}$ имеем интегральное представление δ-функции, соответствующее (I), если

$$\sum_{k=1}^{m} \delta_{\xi^{\bar{\bar{k}}}} (2 \leqslant m < \infty), \quad \sum_{k=-\infty}^{\infty} \delta_{\xi^{\bar{\bar{k}}}} \, (m = \infty)$$

дает интегральное представление (I), то есть дает δ-функцию, сосредоточенную а точке $\xi^1 \equiv \xi \in E_n$.

Рассмотрим на римановой поверхности $R_2^{(m)}$ функцию

$$\dot{y}(\varphi) = \begin{cases} I, & \text{если } |\varphi| < \pi, \\ O, & \text{если } |\varphi| > \pi. \end{cases} \tag{5}$$

Лемма I. Если $2 \leqslant m < \infty$, то на римановой поверхности $R_2^{(m)}$ для функции $\dot{y}(\varphi)$ имеет место интерргральное представление

$$\dot{y}(\varphi) = \frac{1}{m} + \frac{1}{2^m \pi} \int_0^\infty K_m(\beta, \varphi) \, d\beta, \tag{6}$$

где ядро

$$K_m(\beta, \varphi) = \frac{\sin \dfrac{\pi+\varphi}{m}}{\operatorname{ch} \dfrac{\beta}{m} - \cos \dfrac{\pi+\varphi}{m}} + \frac{\sin \dfrac{\pi-\varphi}{m}}{\operatorname{ch} \dfrac{\beta}{m} - \cos \dfrac{\pi-\varphi}{m}}. \tag{7}$$

Доказательство. Так как при любом $\beta \geqslant \beta_0 > 0$

$$K_m(\beta, \varphi) = 2 \sum_{k=1}^{\infty} e^{-\bar{\bar{k}} \frac{\beta}{m}} \left[\sin \frac{\bar{\bar{k}}}{m} (\varphi + \pi) + \sin \frac{\bar{\bar{k}}}{m} (\pi - \varphi) \right], \tag{8}$$

то

$$\int_0^\infty K_m(\beta, \varphi) \, d\beta = 2m \sum_{k=1}^{\infty} \frac{1}{\bar{\bar{k}}} \left[\sin \frac{\bar{\bar{k}}}{m} (\varphi + \pi) + \sin \frac{\bar{\bar{k}}}{m} (\pi - \varphi) \right]. \tag{9}$$

Используя теперь разложение в ряд фурье функции

$$f(x) = \begin{cases} \dfrac{\pi - \bar{\bar{x}}}{2}, & \text{если } \bar{\bar{x}} \in (0, 2\pi), \\ 0, & \text{если } \bar{\bar{x}} \in (0, 2\pi), \end{cases}$$

получаем, что

$$\sum_{k=1}^{8} \frac{1}{\bar{k}} \sin \frac{\pi+\varphi}{m} \bar{k} = \frac{\pi}{2} - \frac{\pi \pm \varphi}{2m},$$

если

$$0 < \frac{\pi}{2} - \frac{\pi \pm \varphi}{2m} < 2\pi.$$

Следовательно, при $|\varphi| < \pi$ имеем

$$\dot{J}(\varphi) = \frac{1}{m} + \frac{2m}{2m\pi}\left(\frac{\pi}{2} - \frac{\pi+\varphi}{2m} + \frac{\pi}{2} - \frac{\pi-\varphi}{2m}\right) = 1,$$

а при $|\varphi| > \varphi$

$$\dot{J}(\varphi) = \frac{1}{m} + \frac{1}{\pi}\left(\pm\frac{\pi}{2} - \frac{\pi+\varphi}{2m} \mp \frac{\pi}{2} - \frac{\pi-\varphi}{2m}\right) = 0.$$

Лемма 2. На римановом многообразии $R_n^{(\infty)}$ для функции $\dot{J}(\varphi)$ имеет место интегральное представление

$$\dot{J}(\varphi) = \frac{1}{\pi}\int_0^\infty K_{(\infty)}(\beta, \varphi) d\beta, \tag{10}$$

где

$$K_{(\infty)}(\beta, \varphi) = \frac{\pi+\varphi}{\beta^2+(\pi+\varphi)^2} + \frac{\pi-\varphi}{\beta^2+(\pi-\varphi)^2}. \tag{11}$$

Доказательство. Если $|\varphi| < \pi$, то

$$\frac{1}{\pi}\int_0^\infty K_{(\infty)}(\beta, \varphi) d\beta = \frac{1}{\pi}\left[\operatorname{arc\,tg} \frac{\beta}{\pi+\varphi} + \operatorname{arc\,tg} \frac{\beta}{\pi-\varphi}\right]_0^{(\infty)} = 1.$$

Пусть $\varphi > \pi$. Положим $\varphi = м + \gamma(\gamma > 0)$, Получим :

$$\frac{1}{\pi}\int_0^\infty K_{(\infty)}(\beta, \varphi) d\beta = \frac{1}{\pi}\left[\operatorname{arc\,tg} \frac{\beta}{2\pi+\lambda} + \operatorname{arc\,tg} \frac{\beta}{\gamma}\right]\Bigg|_0^\infty = 0.$$

Если для $\varphi < -\pi$ положить $\pi = -\pi - \gamma(\gamma > 0)$, то

$$\frac{1}{\pi}\int_0^\infty K_{(\infty)}(\beta, \varphi) d\beta = \frac{1}{\pi}\left[-\operatorname{arc\,tg} \frac{\beta}{\gamma} + \operatorname{arc\,tg} \frac{\beta}{2\pi+\gamma}\right]\Bigg|_0^\infty = 0.$$

Теорема 2. Если функция $f(t, \lambda)$ удовлетворяет условиям теоремы 1, то на римановом многообразии $R_n^{(m)}$ $(2 \leqslant m < \infty$ имеет место интегральное предстамление распределения (δ-функции) Дирака

$$\delta_\xi^{(m)} = \lim_{t \to +0} \frac{w_n}{(2\pi)^n} \int_0^\infty f(t, \lambda) \lambda^{n-1} \Phi_{n,m}(\lambda, R, \varphi - \varphi_0) d\lambda, \tag{12}$$

где

$$\Phi_{n,m}(\lambda, R, \varphi-\varphi_0) = j_{\frac{n-2}{2}}(\lambda R) J(\varphi - \varphi_0) - \frac{1}{2\pi m} \int_0^\infty j_{\frac{n-2}{2}}(\lambda R_1) \times$$

$$\times K_m(\beta, \varphi - \varphi_0) d\beta, \quad R^2 = r^2 + \rho^2 - 2r\rho \cos(\varphi - \varphi_0) + \sum_{k=1}^{n} (x_k - \xi_k)^2,$$

$$R_1^2 = r^2 + \rho^2 + 2r\rho \operatorname{ch} \beta + \sum_{k=3}^{n} (x_k - \xi_k)^2 \cdot$$

Доказательство. То что $\delta_\xi^{(m)}$ дает меру Дирака на римановом многообразии $R_n^{(m)}$ следует из того, что δ-образной последовательности непрерыаных в E_n функций

$$\delta_\xi^{(m)}(\varepsilon) = \frac{w_n}{(2\pi)^n} \int_0^\infty f(\varepsilon, \lambda) \lambda^{n-1} j_{\frac{n-2}{2}}(\lambda R) d\lambda$$

на многообразии $R_n^{(m)}$ соответствует $\delta^{(m)}$ -образная последовательность непрерывных функций

$$\delta_\xi^{(m)}(\varepsilon) = \frac{w_n}{(2\pi)^n} \int_0^\infty f(\varepsilon, \lambda) \lambda^{n-1} \Phi_{n,m}(\lambda, R, \varphi - \varphi_0) d\lambda,$$

что очевидно, если, учитывая (6), ядро $\Phi_{n,m}$ представить в виде :

$$\Phi_{n,m} = \frac{1}{m} \left[j_{\frac{n-2}{2}}(\lambda R) + \frac{1}{2\pi} \int_0^\infty [j_{\frac{n-2}{2}}(\lambda R) - j_{\frac{n-2}{2}}(\lambda R_1)] \times K_m(\beta, \varphi - \varphi_0) d\beta \right]. \tag{13}$$

Покажем, что $\sum_{k=1}^{m} \delta_{\xi^k}^{(m)} = \delta_\xi$. Пусть точка $\xi = \xi^1 \in R_n^{(1)} = R_2^{(1)} \times E_{n-2} \equiv E_n$ и имеет координаты $(\rho, \varphi_0, \xi_3, \ldots, \xi_n$. Тогда точки $\xi^{\bar{k}} \in R_2^{(\bar{k})} \times E_{n-2} = R_n^{(\bar{k})}$ $(\bar{k} = \overline{1, m})$ и будут иметь координаты $(\rho, \varphi_0 + 2(\bar{k}-1)\pi, \xi_3, \ldots, \xi_n)$. Следовательно, имеем :

$$\sum_{\bar{k}=1}^{m} \delta_{\xi^{\bar{k}}}^{(m)} = \lim_{t \to +0} \frac{w_n}{(2\pi)^n} \int_0^\infty f(t, \lambda) \lambda^{n-1} \{ j_{\frac{n-2}{2}}(\lambda R) J(\varphi - \varphi_0) -$$

$$-\frac{1}{2\pi m}\int_0^\infty j_{\frac{n-2}{2}}(\lambda R_1)\sum_{k=1}^m K_m(\beta,\varphi-[\varphi_0+2(k-1)\pi]d\beta\}d\lambda.$$

Из тождества (3)

$$\operatorname{ch}\beta-\cos(\partial-\varphi_0)=2^{m-1}\prod_{\bar{k}-1}^m\left[\operatorname{ch}\frac{\beta}{m}-\cos\frac{\varphi-[\varphi_0+2(k-1)\pi]}{m}\right]$$

путем логарифмического дифферецирования с учетом четности функции $\operatorname{ch} z$ и $\cos z$ получим :

$$\sum_{k=1}^m\frac{1}{m}\frac{\sin\frac{\pi\pm\varphi+[\varphi_0+2(k-1)\pi]}{m}}{\operatorname{ch}\frac{\beta}{m}-\cos\frac{\pi\pm\varphi\mp[(\varphi_0+2(k-1)\pi]}{m}}=\frac{\sin(\pi\pm\varphi\mp\varphi_0}{\operatorname{ch}\beta-\cos(\pi\pm\varphi\mp\varphi_0)}.$$

Поскольку

$$\cos(\pi\pm(\varphi-\varphi_0))=-\cos(\varphi-\varphi_0),\ \sin(\pi\pm(\varphi-\varphi_0))=\pm\sin(\varphi-\varphi_0),$$

то

$$\sum_{k=1}^m K_m(\beta,\varphi-[\varphi_0+2(k-1)\pi]=-\frac{\sin(\varphi-\varphi_0)}{\operatorname{ch}\beta+\cos(\varphi-\varphi_0)}+\frac{\sin(\varphi-\varphi_0)}{\operatorname{ch}\beta+\cos(\varphi-\varphi_0)}\equiv 0\,(*)$$

и

$$\sum_{k=1}^m\delta_{\xi^k}^{(m)}=\lim_{t\to+0}\frac{w_n}{2\pi)^n}\int_0^\infty f(t,\lambda)\,1^{n-1}j_{\frac{n-2}{2}}(\lambda R)d\lambda=\delta_{\xi}.$$

Следствие I. В случае $m=2$ имеем :

$$K_2(\beta,\varphi-\varphi_0)=\frac{2\cos\frac{\varphi-\varphi_0}{2}\operatorname{ch}\frac{\beta}{2}}{\operatorname{sh}^2\frac{\beta}{2}+\cos^2\frac{\partial-\varphi_0}{2}},\ \int_0^\infty K_2(\beta,\varphi-\varphi_0)d\beta=2\pi,$$

$$\Phi_2=j_{\frac{u-2}{2}}(\lambda R)-\frac{1}{\pi}\int_0^\infty j_{\frac{n-2}{2}}(\lambda R_1^*)\frac{d\bar{\bar{u}}}{1+u^2},$$

$$R_1^{*2}=(r+\rho)^2+\sum_{k=3}^n(\bar{\bar{x}}_k-\xi_k)^2+4r\rho\bar{\bar{u}}^2\cos^2\frac{\varphi-\varphi_0}{2}\equiv R^2+q^2(1+\bar{u}^2).$$

Используя известный интеграл (3)

$$\int_{-\infty}^\infty j_{\frac{n-1}{2}}(\lambda\sqrt{R^2+s^2})d\bar{\bar{s}}=\frac{2\pi\omega_n}{\lambda\omega_{n+1}}j_{\frac{n-2}{2}}{}_2(\lambda R),$$

получаем, что

$$\int_0^{\infty} j_{\frac{n-2}{2}}(\lambda R_1^{(*)})\frac{d\bar{u}}{1+\bar{u}^2}=\pi j_{\frac{n-2}{2}}(\lambda R)-\frac{\lambda\omega_{n+1}}{2\omega_n}\int_{-\infty}^{\infty} j_{\frac{n-1}{2}}(\lambda\sqrt{R^2+\bar{s}^2})d\bar{s}.$$

Поэтому

$$\Phi_2=\frac{\lambda\omega_{n+1}}{2\pi\omega_n}\int_{-\infty}^{\infty} j_{\frac{n-1}{2}}(\lambda\sqrt{R^2+\bar{s}^2})\,d\bar{s}$$

и

$$\delta_{\xi}^{(2)}=\lim_{t\to+0}\frac{\omega_{n+1}}{(2\pi)^{n+1}}\int_0^{\infty} f(t,\lambda)\lambda^n\int_{-\infty}^{\infty} j_{\frac{n-1}{2}}(\lambda\sqrt{R^2+\bar{s}^2})d\bar{s}d\lambda, \tag{***}$$

$$\left(q=2\sqrt{r\rho}\cos\frac{\varphi-\varphi_0}{2}\right).$$

Теорема 3. Если функция $f(t,\lambda)$ удовлетворяет условиям теоремы 1, то на римановом многобразии $R_n^{(\infty)}=R_2^{(\infty)}\times E_{n-2}$ имеет место интегральное представление распределения Дирака (меры Дирака)

$$\delta_{\xi}^{(\infty)}=\lim_{t\to+\infty}\frac{\omega_n}{(2\pi)^n}\int_0^{\infty} f(t,\lambda)\lambda^{n-1}\Phi_{n,(\infty)}(\lambda,R,\varphi-\varphi_0)d\lambda, \tag{14}$$

где

$$\Phi_{n,(\infty)}=j_{\frac{n-2}{2}}(\lambda R)\dot{J}(\varphi-\varphi_0)-\frac{1}{\pi}\int_0^{\infty} j_{\frac{n-2}{2}}(\lambda R_1)K_{(\infty)}(\beta,\varphi-\varphi_0)\,d\beta.$$

Доказательство. Будем считать, что множество точек лежит на первом экземпляре риманового многообразия $R_n^{(\infty)}$, если $i\varphi-\varphi_0|<\pi$, на $\bar{\bar{k}}$ к-ом ($\bar{\bar{k}}>0$), если $(2\bar{\bar{k}}-1)\pi<\varphi-\varphi_0<(2\bar{\bar{k}}+1)\pi$, на $(-k)$ -ом, если $-(2\bar{\bar{k}}+1)\pi<\varphi-\varphi_0<-(2\bar{\bar{k}}-1)\pi$. Покажем, что $\sum_{k=-\infty}^{\infty}\delta_{\xi_k}^{(\infty)}=\delta_{\xi}$. Так как из тождества (3)

$$\operatorname{ch}\beta-\cos(\partial-\varphi_0)=\frac{(\varphi-\varphi_0)^2+\beta^2}{2}\prod_{\bar{\bar{k}}=-\infty}^{\infty}\frac{\beta^2+(\varphi-\varphi_0-2k\pi)^2}{4K^2\pi^2},$$

путем логарифмического дифференцирования получаем, что

$$\frac{\sin(\varphi-\varphi_0)}{2[\operatorname{ch}\beta-\cos(\varphi-\varphi_0)]}=\sum_{\bar{\bar{k}}=-\infty}^{\infty}\frac{\varphi-\varphi_0-2\bar{\bar{k}}\pi}{\beta^2+(\varphi-\varphi_0-2\bar{\bar{k}}\pi)^2},$$

то

$$\sum_{\bar{k}=-\infty}^{\infty} K_{(\infty)}(\beta,\, \partial-\varphi_0-2k\pi)=\frac{1}{2}\left[\frac{\sin(\pi+\partial-\varphi_0)}{\operatorname{ch}\beta-\cos(\pi+\varphi-\gamma_0)}+\right.$$

$$\left.+\frac{\sin(\pi-\varphi+\varphi_0)_G}{\operatorname{ch}\beta-\cos(\pi--+\varphi_0)}\right]=0 \qquad (**)$$

и, следовательно,

$$\sum_{\bar{k}=-\infty}^{\infty} \delta_{\xi}^{(\infty)}=\lim_{t\to+\infty}\frac{\omega_n}{(2\pi)^n}\int_0^{\infty} f(t,\lambda)\lambda^{n-1}j_{\frac{n-2}{2}}(\lambda R)d\lambda=\delta_{\xi}.$$

То, что $\delta_{\xi}^{(\infty)}$ опеделяет мету Дирака на $R_n^{(\infty)}$ следует из того факта- что δ-образной последовательности непрерывных функций $\delta_{\xi}(\varepsilon)$ на многообразии $R_n^{(\infty)}$ соответствует δ-образная последовательность пепрерыв, ных функций

$$\delta_{\xi}^{(\infty)}(\varepsilon)=\frac{\omega_n}{(2\pi)^n}\int_0^{\infty} f(\varepsilon,\lambda)\lambda^{n-1}\Phi_{n,(\infty)}(\lambda,\, R,\, \varphi-\varphi_0)d\lambda,$$

что очевидно, если, учитывая (10), ядро $\Phi_{n,(\infty)}$ представить в виде

$$\Phi_{n,(\infty)}=\frac{1}{\pi}\int_0^{\infty}[j_{\frac{n-2}{2}}(\lambda R)-j_{\frac{n-2}{2}}2(\lambda R_1)]K_{(\infty)}(\beta,\, \varphi-\varphi_0)d\beta.$$

Применим интегральные представления (12) и (14) для построения фундаментальных решений задачи Коши на римановых многообразиях

$$R_{n+1}^{+(m)}=[0,\infty)\times R_n^{(m)}=[0,\infty)\times R_2^{(m)}\times E_{n-2}=R_2^{(m)}\times E_{n-1}^{+}(m\geqslant 2)$$

и $R_{n+1}^{+(\infty)}=R_2^{(\infty)}\times E_{n-1}^{+}$, где E_{n-}^{+} — эвклидовое полупространство точек $(t,\bar{\bar{x}}_1,\bar{\bar{x}}_3,\ldots,\bar{\bar{x}}_n)$, удовлетворяющих неравенствам $t>0,-\infty<\bar{\bar{x}}_j<+\infty(j=$ $=1,2,\ldots,n)$, для параболического уравнения, инвариантного по переменным $(\bar{\bar{x}}_1,\bar{\bar{x}}_2,\ldots,\bar{\bar{x}}_n)$ относительно вращений вокруг начала координат пространства E_n :

$$A\left(\Delta,\frac{\partial}{\partial t}\right)\bar{\bar{u}}=\sum_{k=0}^{l} A_k(\Delta)\frac{\partial^{\bar{k}}\bar{\bar{u}}}{\partial t^{\bar{k}}}=0, \qquad (15)$$

где $A_l=I$, $\Delta=\sum_{\bar{k}=1}^{n}\frac{\partial^2}{\partial\bar{\bar{x}}_k^2}$, $A_{\bar{k}}(\bar{\bar{z}})$ — функции представимы всюду сходящимися в комплексной $\bar{\bar{z}}$-плоскости рядами.

Определение 2. Оператор $A\left(\Delta,\frac{\partial}{\partial t}\right)$ называется параболическим, если при достаточно больших $\lambda\in E_1$ все корни характеристического уравнения

$$F(\bar{z}, -\lambda^2) = \sum_{k=0}^{l} A_{\bar{k}}(-\lambda^2) z^{\bar{k}} = 0 \tag{16}$$

имеют отрицательные действительные части и удовлетворяют оценкам

$$|\mathrm{Re}\, \bar{z}_k| > c\lambda^{2\alpha}, \quad c \geqslant c_0 > 0, \quad d \geqslant d_0 > 0. \tag{17}$$

Опеределение 3. m-разветвленным (бесконечно разветвленным) фундаментальным решением задачи Коши с гиперплоскостью ветвления $(0, 0)\times E^{+}_{n-1}$ для параболического оператора A называется такое фундаментальное решение оператора A $G_m(t, \bar{\bar{x}}, \xi)(G_{(\infty)}(t, \bar{\bar{x}}, \xi))$ которое в $\overset{+}{R}{}^{(m)}_{n+1}$ $(\overset{+}{R}{}^{(\infty)}_{n+1})$ определено всюду, кроме гиперплоскости, ветвления и удовлетворяет таким свойствам :

a) $G_m(t, \bar{\bar{x}}, \xi)(G_{(\infty)}(t, \bar{\bar{x}}, \xi)$ имеет в $\overset{+}{R}{}^{(m)}_{n+1}$ $(\overset{+}{R}{}^{(\infty)}_{n+1})$ одну характеристическую особенность в точке $(0, \xi) \in \overset{+}{R}{}^{(1)}_{n+1} \equiv E^{+}_{n+1}$;

b) $G_m(t, \bar{\bar{x}}, \xi)(G_{(\infty)}(t, \bar{\bar{x}}, \xi)$ бесконечно дифференцируемая всюду, кроме точки $(0, \xi) \in E^{+}_{n+1}$ и гиперплоскости ветвления ;

b) $\sum\limits_{k=1}^{m} G_m(t, \bar{\bar{x}}, \xi^k) = G_{(\infty)}(t, \bar{\bar{x}}, \xi) \left(\sum\limits_{k=-\infty}^{\infty} G_{(\infty)}(t, \bar{\bar{x}}, \xi^k) = G(t\ \bar{\bar{x}}, \xi) \right)$, где $(t, \xi) \equiv (t, \xi)$, а точки $(t, \xi^k) \in \overset{+}{R}{}^{(k)}_{n+1}$ лехат на том же месте, что и (t, ξ), но в экземплярах $R^{(k)}_{n+1}$,$G(t, x, \xi)$ — обычное фундаментальное решение задачи Коши для оператора A.

Примечания. a. В следующей работе авторов, при тех же обозначениях и с продолжением нумерации формул, лемм, теореС и опеделений будут изложены результаты о структуре фундаментальных решений задачи Коши для инвариантных параболических операторов нй римановых многообразиях.

б. Настоящая работа посвящается памяти Б. Римана (Bernhard RIEMANN), основоположника многих ветвей современной и будущей Математики, в предверии 150-и летия — со дня рождения.

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SOLUȚIILE FUNDAMENTALE PE VARIETĂȚI RIEMANNIENE REFERITOARE LA PROBLEMA LUI CAUCHY PENTRU OPERATORI PARABOLICI INVARIANȚI

(Rezumat)

Autorii studiază probleme mixte referitoare la operatori parabolici cu frontieră non netedă. În cadrul acestui studiu se construiesc pe varietăți riemanniene speciale $R^{(m)}_n = R^{(m)}_2 \times E_{n-2}$ $(2 \leqslant m \leqslant \infty)$, unde $R^{(m)}_2$ și, respectiv $R^{(\infty)}_2$ sînt suprafețele lui Riemann ale funcțiilor

$$\bar{\bar{w}} = \sqrt[m]{\bar{z}} (z = \bar{\bar{x}}_1 + i\bar{\bar{x}}_2) \text{ și } w = \ln z = \lim_{m \to \infty} m \left(\sqrt[m]{\bar{\bar{z}}} - 1\right),$$

ar E_k este spațiu euclidian cu k dimensiuni, soluțiile fundamentale referitoare la problema lui Cauchy pentru operatori parabolici invarianți. Unealta principală în construcția acestor soluții fundamentale este reprezentarea integrală a funcției generalizate a lui Dirac (funcției τ). Se enunță și se demonstrează 3 teoreme, 2 Leme și se introduc trei noi definiții utile, pe cînd problema structurii unor atare soluții va fi discutată într-o lucrare separată, programată să apară în același BULETIN.

Această lucrare este dedicată Memoriei lui Bernhard RIEMANN, fondatorul unor ramuri de seamă ale Matematicii de azi și de miine, în pragul împlinirii a 150 de ani de la nașterea sa.

U.D.C. 517.944

FUNDAMENTAL SOLUTIONS ON RIEMANN-SURFACES OF THE CAUCHY PROBLEM CONCERNING INVARIANT PARABOLIC OPERATORS

(Summary)

A Study of mixed problems for parabolic operators on non smooth frontier is exposed. In the framework of this study fundamental solutions of the Cauchy problem on special Riemann-spaces (special Riemann varieties) $R^{(m)}_n = R^{(m)}_2 \times E_{n-2}(2 \leqslant m \leqslant \infty)$ are constructed where $R^{(m)}_2$ and $R^{(m)}_2$ are Riemann — surfaces of the functions

$$\bar{w} = \left(\sqrt[m]{z} \, (z = \bar{\bar{x}}_1 + i\bar{\bar{x}}_2) \text{ and } w = \ln \bar{\bar{z}} = \lim_{m \to \infty} m \left(\sqrt[m]{\bar{\bar{z}}} - 1\right),\right.$$

respectively, while E_k is a k-dimensional euclidean space. The principal tool in construction of the corresponding fundamental solutions is the integral representation of the Dirac generalized function δ. There are to be found here 3 theorems with their proves, 2 lemmas and three new definitions, while a study concerning the structure of these fundamental solutions will appear seperately in the same BULLETIN.

The paper is dedicated to Bernhard RIEMANN, the founder of some of the fundamental branches of Mathematics, on the treshold of the 150 th anniversary of his birthday.

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C.Z.U. 527.52

GRID INFLUENCE UPON PLASMA'S PROPER OSCILLATORY CIRCUIT PARAMETERS IN A CONTINUOUS CURRENT GLOW DISCHARGE

BY

A. NAT*)

As shown in [1], under certain conditions, glow discharge in continuous current withont positive column constitutes an electric oscillation generator whose equivalent oscillation circuit is shown in Fig. 1, where R_N is negative resistance situated between the inner wall of lightning tube and anode plate ; C is capacity between the inner wall of lightning tube and mass ; L is inductance with its true resistance R_L ; C_f is filtration capacity.

The paper's pourpose is to localise some elements of the oscillatory circuit and to find their influencing factors by means of experimental data.

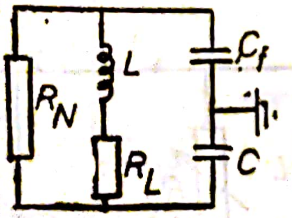

Fig. 1

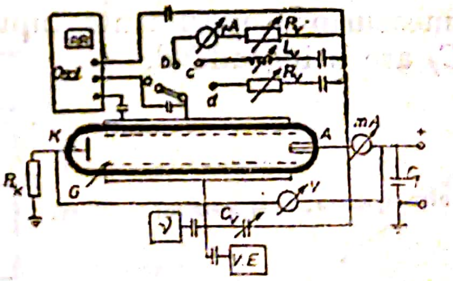

Fig. 2

To this aim the experimental device in Fig. 2 has been used; its anode plate is made of a 1 mm diameter platinum wire ; its cathode is a 40 mm aluminium plate. On the outer surface of lightning tube there is a metalic foil with a length equal to the distance between cathode and anode. Inside the tube a cylindric, metalic grid has been introduced ; it is coaxial to lightning tube, and placed at a 2 mm distance from the tube's wall ; this distance is a proof that there is no direct contàct between grid and tube's-inner wall. The grid is connected to anode plate by a linking capacity C_{k-g} anod a variable resistance R_v (Fig. 2 position a), or a variable inductance, L_v (Fig. 2 position b). By modifying R_v, L_v and C_v it becomes obvious

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that oscillation frequency depends on the values of C_v, when L_v and R_v are constant, and on the values of L_v or R_v, when C_v is constant. The experimental results obtained are shown in Fig. 3, 4, 5. In order to explain these data various equivalent circuits have been considered and resonance frequency has been calculated. The only equivalent circuit which verifies all data is the one in Fig. 6 when R_v is in outer in circuit, grid-anode, and the one in Fig. 7 when L_v is in outer circuit, grid-anode, on condition that resistance R_L of inductance is much smaller than inductive reactance ωL.

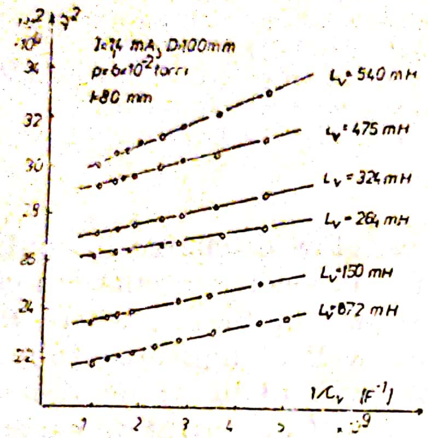

Fig. 3

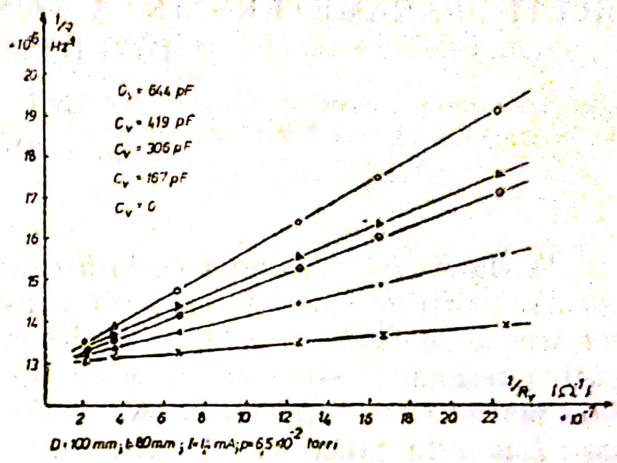

Fig. 4

In this case the region inner wall-grid behaves like a resistance R and, C_p is inner wall-outer wall capacity; C_m is outer wall — mass capacity, to which measuring instruments input capacity, variable C_v and filtration capacity C_f are being added.

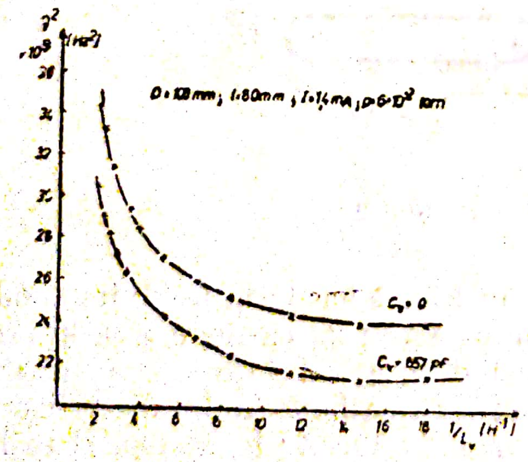

Fig. 5

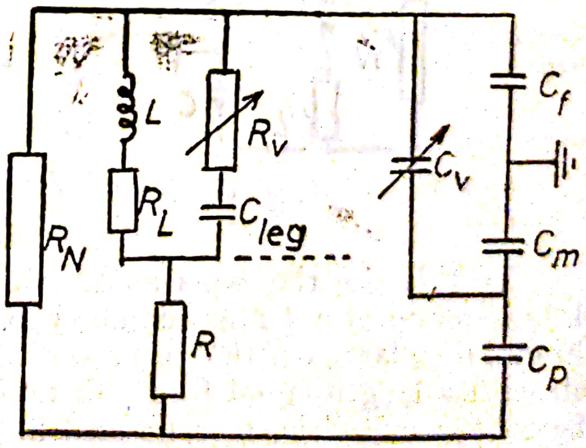

Fig. 6

Considering the equivalent circuit in Fig. 6, the following formula is obtained for frequency:

$$\omega^2 = \frac{R_v^2\left(\frac{L}{C} - R^2\right)}{L^2(R_v^2 + R)^2} \tag{1}$$

On condition that $\frac{L}{C} \gg R^2$, relation (1) becomes:

(2) $$\frac{1}{\omega} = \sqrt{LC} + R\sqrt{LC}\,\frac{1}{R_v}$$

where

$$\frac{1}{C} = \frac{1}{C_p} + \frac{1}{C_m + C_v}$$

For C_v=const., relation (2) represents a straight line of the form:

$$Y = A + BX,$$

where $Y = \frac{1}{\omega}$, $A = \sqrt{LC}$; $B = RA$ and $X = \frac{1}{R_v}$. The fact that relation (2) represents a straight line is acknowledged by the data in Fig. 4 and by statistical processing of a great number of measurements which indicate an empirical correlation factor to be found in $0{,}91 \leqslant r \leqslant 0.95$ range, with a higher than 0.99 degree of reliability. By statistically calculating the A and B coefficients belonging to the family of straight lines obtained for different values of C_v, it is found that,

$$R = \frac{B}{A},$$

are values which increase lineary with the increase of C_v (Fig. 8); this offers information on the proper oscillatory circuit.

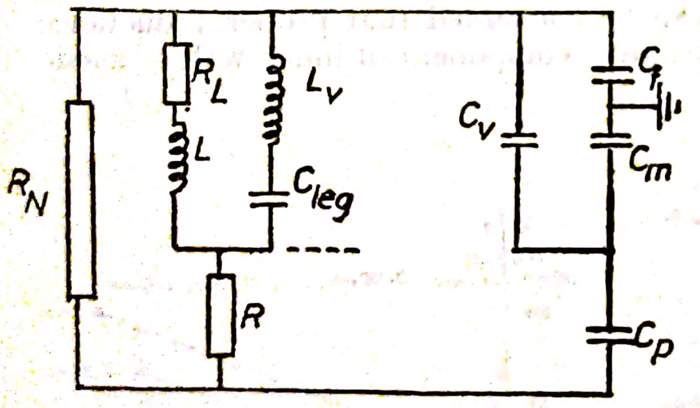

Fig. 7

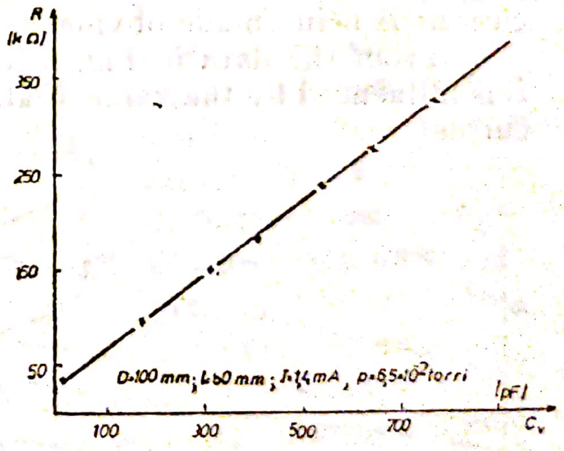

Fig. 8

The calculation of resonance frequency of the equivalent circuit in Fig. 8 leads to:

(3) $$\omega^2 = \frac{1}{C_p}\left(\frac{1}{L} + \frac{1}{L_v}\right) - \frac{R^2}{L_v^2} + \left(\frac{1}{L} + \frac{1}{L_v}\right)\frac{1}{C_m + C_v}$$

If the value of C_m is negligible in comparison with C_v value, or it is considered included in C_v value, for L_v=const., relation (3) is the equation of a straight line, of the form:

$$Y' = A' + B'X'$$

where :

$$Y'=\omega^2,\ X'=\frac{1}{C_v}\ ,\ A'=\frac{1}{C_v}B'-\frac{R^2}{L_v^2}\ ;\ B'=\frac{1}{L}+\frac{1}{L_v}\ ,$$

as data in Fig. 3 demonstrate.

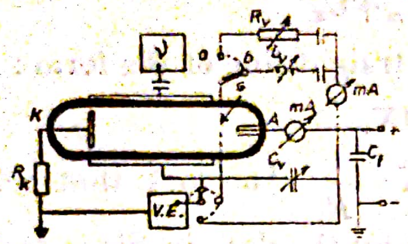

Fig. 9

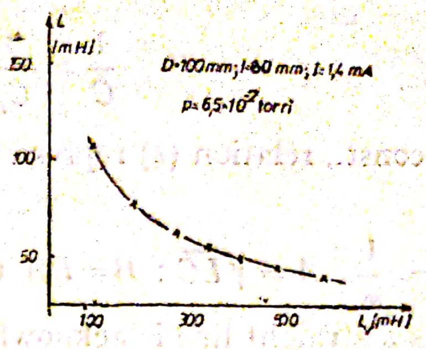

Fig. 10

By statistic calculation of B' value for different L_v values the results shown in Fig. 9 are obtained ; it is obvious that L depends on L_v value, fact that may be explained as follows : by increasing L_v inductive reactance of outer circuit grid-anode also increases and, at the same time, a diminishing of alternating current in outer circuit grid-anode also occurs. As a consequence, the alternative component of inner circuit, inner wall-anode increases with the L_v increase and this fact causes the decrease of L. This conclusion agrees with the results shown in [2], where a similar dependence of proper inductance on the value of intensity of discharge in continuous current is being made obvious.

From the data in Fig. 9 it can be concluded that proper inductance L is influenced by the value of alternative component of inner wall — anode current.

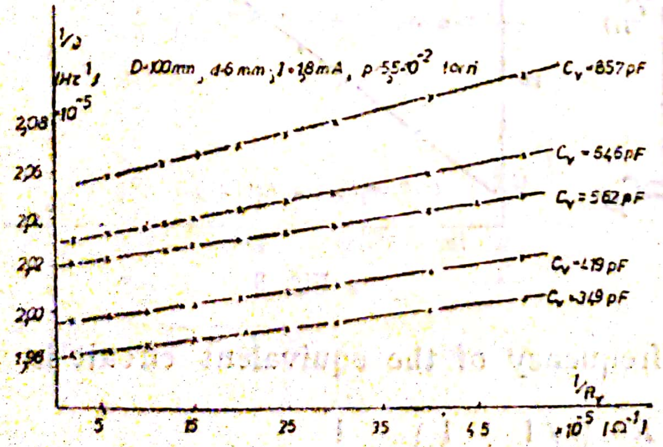

Fig. 11

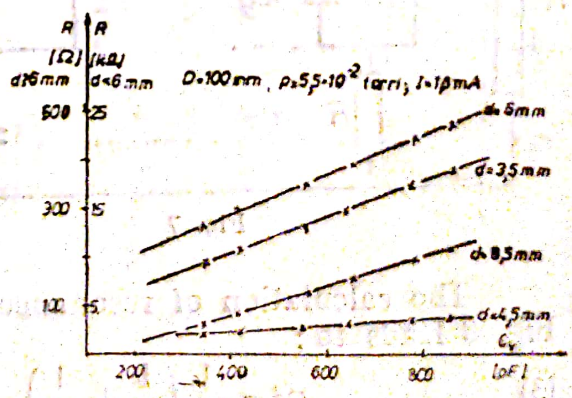

Fig. 12

The dependance of L on the value of L_v explains why $\nu^2=f\left(\frac{1}{L_v}\right)$, for $C_v=$const. presents an inward concavity (Fig. 5) and not an outward one as could be obtained from (3) if L weren't equal to $f(L_v)$.

Thus it is obvious that the region inner wall-grid has a behaviour equivalent to a resistance which depends on the value of the variable capacity C_v and, proper inductance L depends on the alternative component of electron current inner wall-anode.

For furter checking, in the lightning tube there has been introduced a metalic foil equal in length to the distance anode-cathode, stuck to the inner surface of the tube.

The laggings between the signals given by the inner wall and grid have been measured and the result is that the two signals are in phase, fact that proves that between inner wall and grid there are no elements which can cause laggings.

These measurements have been made in argon and air and no qualitative differences have been noticed. For a complete elucidation of grid influence upon plasma's proper oscilatory circuit parameters in glow discharge and for establishing the space in which the region inner wall-grid behaves like a resistance whose value depends on the alternative current which passes through in the same measurements as those described above have been made but, this time, the device in Fig. 10 has been used. It is different from the one in Fig. 3 by the fact that grid is circular, with a 15 mm diameter and it is placed outside anode. The distance anode-grid can be modified from outside, so that the distance anode-cathode could remain constant. The experimental data obtained and shown in Fig. 11 and 12 have acknowledged the equivalent circuits in Fig. 6 and 7 for bigger than 2 mm distances grid-anode. For distances smaller than 2 mm, under the same experimental conditions, the discharge disappears.

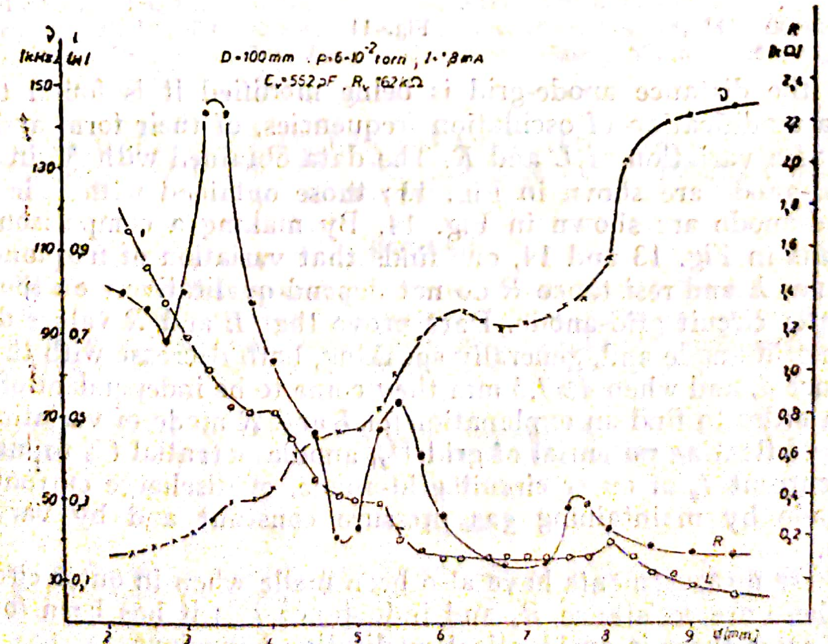

Fig. 13

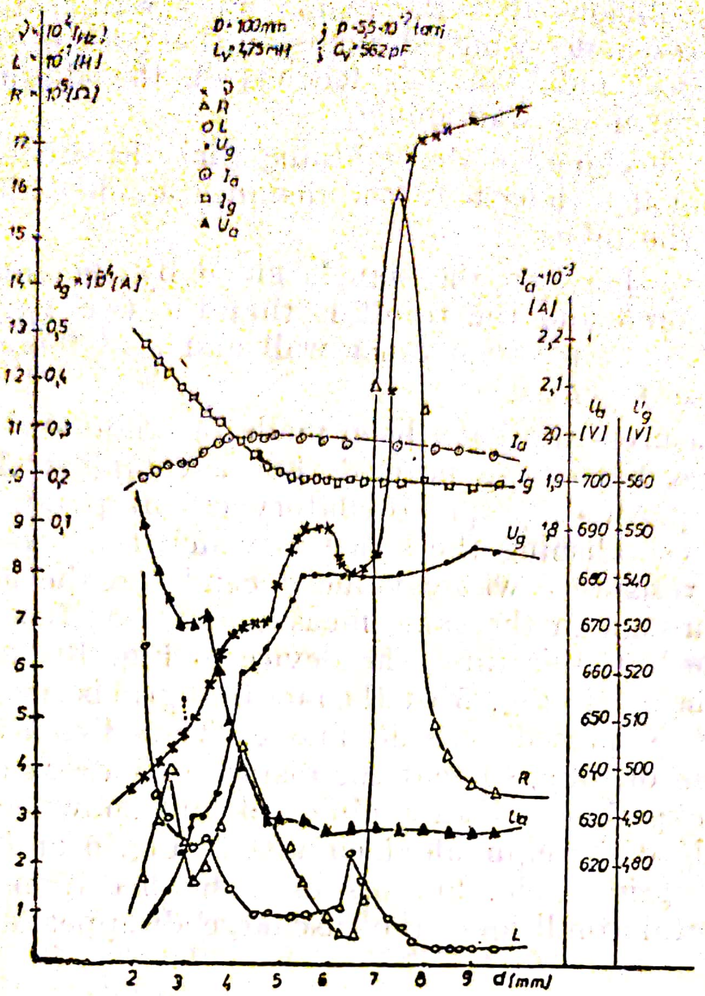

Fig. 14

If the distance anode-grid is being modified it is found that there occurs a modification of oscillation frequencies, of their form and stability as well as a variation of L and R. The data obtained with R_v in outer circuit grid-anode are shovn in Fig. 13; those obtained with L_v in outer circuit grid-anode are shown in Fig. 14. By making a comparison between the results in Fig. 13 and 14, one finds that variation of frequency proper inductance L and resistance R do not depend qualitatively on the elements from outer circuit grid-anode. Data prove that L and R values depend on distance grid-anode and, generally speaking, both decrease with the increase of distance d, and when $d \geqslant 7.5$ mm they come to be independent of d.

In order to find an explanation for L and R mode of variation measurements of floating potential of grid U_g, anodic potential U_a to mass, alternating current I_g of outer circuit grid-anode, of discharge current I_a have been made by maintaining gas pressure constant and by varying grid position.

These measurements have also been made when in outer circuit grid-anode there are resistance R_v and inductance L_v; it has been found that in both cases there is an identical qualitative behaviour.

To explain the experimental data it must be taken into account that grid is galvanically insulated from anode and that it behaves like an elec-

trically insulated electrode in contact with plasma which generates a negative potential hole as compared to plasma potential.

The depth of potential hole depends on density of space charges existing in that region of plasma and having grid form [3], [4].

The depth of potential hole influences anode capacity of trapping secondary electrons emitted by wall, fact that leads to grid influencing both discharge and proper oscillatory circuit parameters. Some secondary phenomena must also be taken into consideration, such as recombining of secondary, electrons near grid, their rejection by grid, supplementary ionization in region grid-anode. One can notice that the difference of potential U_a corresponding to R maximums is of 15 V, 28 V, 41 V, values which may be accorded to ionization potentials for argon.

From the above data the following conclusions can be drawn: — region grid — inner wall behaves like a resistance whose value depends on alternative component of inner wall-anode current.

— inductance L of oscillatory circuit is concentrated between grid and anode, at a short distance from anode.

— proper inductance L is influenced by àlternative component of inner wall-anode current.

— resistance R dependens on secondary phenomena which take place in plasma ; R is probably influenced by existing space charges.

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INFLUENŢA UNEI GRILE ASUPRA PARAMETRILOR CIRCUITULUI OSCILLANT PROPRIU AL PLASMEI DINTR-O DESCĂRCARE LUMINESCENTĂ ÎN CURENT CONTINUU

(Rezumat)

În lucrare s-a studiat, experimental, modul în care pot fi influenţaţi parametrii circuitului oscilant propriu al unei descărcări luminescente prin introducerea în tubul de descărcare a unei grile metalice. Prin această metodă s-a obţinut o localizare în spaţiul descărcării a rezistenţei şi a inductanţei proprii şi s-au stabilit factorii care le influenţează.

L'INFLUENCE D'UNE GRILLE SUR LES PARAMETRES DU CIRCUIT OSCILLANT PROPRE DU PLASMA D'UNE DECHARGE LUMINESCENTE EN COURANT CONTINU

(Resumé)

Dans ce travail de recherche on a étudié, expérimentalement, la manière dans laquelle peuvent être influencés les paramètres du circuit oscillant propre à une décharge luminiscente par l'introduction dans le tube de décharge d'une grille métallique. Par cette méthode on a obtenu une localisation dans l'espace de la décharge, de la résistance et de l'inductance propres et on a établi les facteurs qui les influencent.

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L'ETUDE DES EQUATIONS INTÉGRO-DIFFERENTIELLES DANS L'ESPACE DES DISTRIBUTIONS

I. N. CONSTANTINESCU, GR. N. TĂTARU*

Nous considérons l'équation intégro-différentielle ayant la forme :

(I) $$P\left(\frac{d}{dx}\right)\cdot y(x)+\nu\int_0^x K(x,t)\cdot Q\left(\frac{d}{dt}\right)\cdot y(t)\cdot dt=f(x),\ x\geqslant 0,$$ où l'on suppose :

(1) $$P\left(\frac{d}{dx}\right)=a_0\frac{d^n}{dx^n}+a_1\frac{d^{n-1}}{dx^{n-1}}+\ldots+a_{n-1}\frac{d}{dx}+a^n,\ a_ia_1\in R(i=0,\,1\ldots n)$$

(2) $$Q\left(\frac{d}{dx}\right)=b_0\frac{d^m}{dx^m}+b_1\frac{d^{m-1}}{dx^{m-1}}+\ldots+b_{m-1}\frac{d}{dx}+b_m,\ b_i\in R(i=0,\,1,\ldots m)$$

(3) $$m\leqslant n$$

(4) $$y(x)\in C^n([0,\infty)),\ K(x,t)=K(x-t),\ K(x-t)=0 \text{ pour } t\geqslant x\geqslant 0$$

et localement intégrable dans $[0,+\infty)$

(5) $$\nu=1$$

Dans l'équation (1) $y(x)$ représente la fonction inconnue mais les fonctions $f(x)$ et $K(x)$ sont données. Dans ce qui suit nous nous proposons la réduction de l'équation (1) à l'équation de convolution dans $D'_+(R)$. Pour cela nous nous proposons de déterminer la solution de l'équation :

(1') $$P\left(\frac{d}{dx}\right)\cdot y(x)+\int_0^x K(x-t)\cdot Q\left(\frac{d}{dt}\right)\cdot y(t)\,dt=f(x),\ x\geqslant 0,$$

qui vérifie les conditions initiales :

II. $$y^{(i)}(0)=y_0^i,\ i=0,\,1,\,2,\ldots,n-1$$

Dans ce point on introduit les fonctions : $\bar{f}(x)=\theta(x)\cdot f(x)$; $\bar{K}(x)=\theta(x)\cdot K(x)$; $\bar{y}(x)_F=\theta(x)\cdot y(x)$. On attache, aux équations ainsi définies,

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les distributions régulières correspondentes pour lesquelles nous avons les notations $\overline{f}, \overline{K}, \overline{y}$ Ainsi l'équation (1') devient

$$(1'') \qquad \widetilde{P}\left(\frac{d}{dx}\right) \cdot \overline{y} + \int_0^x \overline{K}(x-t) \cdot \overline{Q}\left(\frac{d}{dt}\right) \cdot \overline{y}(t) \,.\, dt = \overline{f}(x),$$

où les polynômes différentie's : $P\left(\frac{d}{dx}\right)$, $Q\left(\frac{d}{dx}\right)$, sont notés par $\overline{P}\left(\frac{d}{dx}\right)$, $\overline{Q}\left(\frac{d}{dx}\right)$, en indiquant, de cette manière, que les dérivées de la fonction inconnue $y(x)$ répésentent les distributions régulières qui sont générés des dérivées de la fonction inconnue $y(x)$ au sens classique. En utilisant la définition du produit de convolution des distributions régulières, le support desquelles se situe sur $[0,+\infty)$, nous constatons que (1'') devient :

$$(1''') \qquad \widetilde{P}\left(\frac{d}{dx}\right) \cdot \overline{y}(x) + K(x) \cdot Q\left(\frac{d}{dx}\right) \cdot \overline{y}(x) = \overline{f}(x)$$

La fonction $\overline{y}(x) = \theta(x), y(x)$, et ses dérivées jusqu'à l'ordre n, conformement aux conditions initiales (II), pussédent à l'origine une discontinuité de prémière èspece et donc nous avons— les relations suivantes :

$$\frac{\tilde{d}^0}{dx^0} \cdot \overline{y}(x) = \frac{d^0}{dx^0} \overline{y}(x)$$

$$\frac{\tilde{d}}{dx} \overline{y}(x) = \frac{d}{dx} \overline{y}(x) - y_0^0 \delta(x)$$

$$\frac{\tilde{d}^2}{dx^2} \overline{y}(x) = \frac{d^2}{dx^2} \overline{y}(x) - y_0' \delta(x) - y_0^0 \delta'(x)$$

III

. . . .

$$\frac{\tilde{d}^{n-1}}{dx^{n-1}} \overline{y}(x) = \frac{d^{n-1}}{dx^{n-1}} \overline{y}(x) - y_0^{(n-2)} \,.\, \delta(x) - y_0^{(n-3)} \delta'(x) - \ldots$$

$$\ldots - y_0' \; \delta_{(x)}^{(n-3)} - y_0^0 \delta_{(x)}^{(n-2)}$$

$$\frac{\tilde{d}^n}{dx^n} \overline{y}(x) = \frac{d^n}{dx^n} \overline{y}(x) - y_0^{(n-1)} \,.\, \delta(x) - y_0^{(n-2)} \,.\, \delta'_{(x)} - \ldots - y_0^0 \,.\, \delta_{(x)}^{(n-1)}$$

En multipliant les relations (III) respectivement par $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ et en additionant on obtient :

$$\overline{P}\left(\frac{d}{dx}\right) \cdot \overline{y}(x) = P\left(\frac{d}{dx}\right) \cdot \overline{y}(x) - [a_0 \,.\, y_0^0 \,.\, \delta_{(x)}^{(n-1)} + (a_1 \,.\, y_0 + a_0 \; y_0') \delta_{(x)}^{(n-2)} +$$

$$+ (a_2 \,.\, y_0^0 + a_1 \,.\, y_0' + a_0 \,.\, y_0^2) \,.\, \delta_{(x)}^{(n-3)} + \ldots + (a_{n-2} \,.\, y_0^0 + a_{n-3} \,.\, y_0' + \ldots$$

$$\ldots + a_1 \,.\, y_0^{(n-3)}) \,.\, \delta'(x) + (a_{n-1} \,.\, y_0^0 + a_{n-2} \,.\, y_0' + a_{n-3} \,.\, y_0^2 + \ldots + a_1 \,.\, y_0^{(n-2)} + a_0 \,.\, y_{(x)}^{(n-1)}) \delta(x)]$$

En observant les propriétés des indices du deuxième membre on constate que l'égalité antérieure devient :

$$\text{IV} \qquad \widetilde{P}\left(\frac{d}{dx}\right)\cdot\overline{y}(x)=P\left(\frac{d}{dx}\right)\cdot\overline{y}(x)-\sum_{k=0}^{n-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{n-1} a_i\cdot y_0^j\right)\cdot\delta_{(x)}^{(n-k-1)}$$

Observation : Dans cette égalité $P\left(\frac{d}{dx}\right)$ exprime le polynôme différentiel dans le sens de la théorie des distributions. De même on obtient, pour $\overline{Q}\left(\frac{d}{dx}\right)\cdot\overline{y}(x)$, l'expression :

$$\text{V} \qquad \widetilde{Q}\left(\frac{d}{dx}\right)\cdot\overline{y}(x)=Q\left(\frac{d}{dx}\right)\cdot\overline{y}(x)-\sum_{k=0}^{m-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{m-1} b_i\cdot y_0^j\right)\cdot\overline{K}_{(x)}^{(m-k-1)}$$

En utilisant les expressions de $\widetilde{P}\left(\frac{d}{dx}\right)\cdot\overline{y}(x)$ et $\widetilde{Q}\left(\frac{d}{dx}\right)\cdot\overline{y}(x)$ données par (IV), (V), l'équation ($1'''$) devient :

$$(1^{\text{IV}}) \qquad P\left(\frac{d}{dx}\right)\cdot\overline{y}(x)+K(x)*Q\left(\frac{d}{dx}\right)\cdot\overline{y}(x)=\overline{f}(x)+\sum_{k=0}^{n-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{n-1} a_i\cdot y_0^j\right)\delta_{(x)}^{(n-k-1)}+$$

$$+\sum_{k=0}^{m-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{m-1} b_i\cdot y_0^j\right)\cdot\overline{K}_{(x)}^{(m-k-1)}$$

En observant que : $P\left(\frac{d}{dx}\right)\cdot\overline{y}(x)=P\left(\frac{d}{dx}\right)\cdot\delta(x)*\cdot\overline{y}(x)$ et $\overline{K}(x)*Q\left(\frac{d}{dx}\right)\cdot$

$\cdot\overline{y}(x)=Q\left(\frac{d}{dx}\right)\cdot\overline{K}(x)\cdot\overline{y}(x)$, l'équation ($1^{\text{IV}}$) devient :

$$(1^{\text{V}}) \qquad \left[P\left(\frac{d}{dx}\right)\cdot\delta(x)+Q\left(\frac{d}{dx}\right)\cdot\overline{K}(x)\right]*\cdot y(x)=\overline{f}(x)+\sum_{k=0}^{n-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{n-1} a_i\cdot y_0^j\right)\cdot$$

$$\cdot\,\delta_{(x)}^{(n-k-1)}+\sum_{k=0}^{m-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{m-1} b_i\cdot y_0^j\right)\overline{K}_{(x)}^{(m-k-1)}$$

L'équation (1^{V}) corespond, dans les distributions, au problème de Cauchy (II) pour l'équation ($1'$) et represénte une équation de convolution dans $D'_+(R)$.

Pour la détermination de la solution unique $\overline{y}(x)\in D'_+(R)$, on utilise la méthode appliquée aux équations de convolution. La solution fondamentale (1^{V}) est obtenue en appliquant la transformée de Laplace, de l'équation :

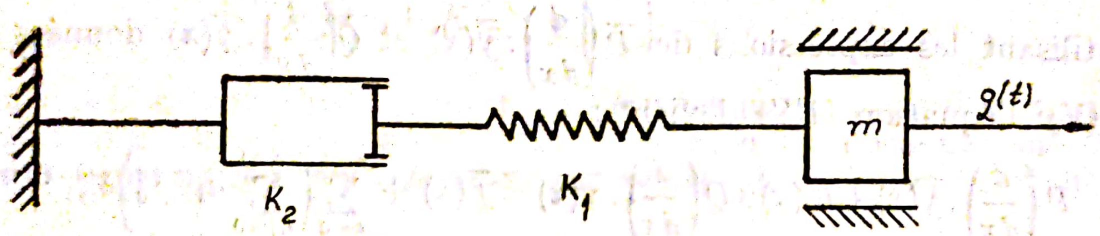
$$\text{(VI)} \qquad L[\overline{F}(x)] = \frac{1}{L\left[P\left(\frac{d}{dx}\right) . \delta(x) + Q\left(\frac{d}{dx}\right) . \overline{K}(x)\right]}$$

Alors la solution de l'équation (1^{V}) est donnée par la relation :

$$\overline{y}(x) = \overline{F}(x) * b(x), \text{ où :}$$

$$b(x) = \overline{f}(x) + \sum_{k=0}^{n-1}\left[\sum_{\substack{i,j=0\\ i+j=k}}^{n-1} a_i . y_0^j\right] . \delta_{(x)}^{(n-k-1)} + \sum_{k=0}^{m-1}\left[\sum_{\substack{i,j=0\\ i+j=0}}^{m-1} b_i . y_0^j\right] \overline{K}_{(x)}^{(m-k-1}, \in D'_+(R)$$

Applications de la théorie des distributions à l'étude du MOUVEMENT DU MODÈLE Maxwell. Dans ce modèle sur le corps de masse m s'exercite la force d'un ressort de constante élastique K_1 et aussi d'un amortisseur de constante de viscosité K_2, une force perturbatrice $q(t)$. La force $q(t)$ entraînne le corps de la position d'équilibre et entretient le mouvement. L'amortisseur et le ressort sont couplés en série.



Dans le moment t quand les deplacements du ressort et du amortisseur sont x_1 et x_2, le corps de masse m est déplacé avec $x = x_1 + x_2$. Sur le corps actionnent les forces :

— La force élastique $F = -K_1 . x_1$, qui s'oppose au mouvement;

— La force perturbatrice qui entraînne le corps de la position de l'équilibre et entretienne le mouvement ;

— La force de résistence $F_e = -K_2 . \dot{x}_2$.

La résultante $R_1(t)$ de ce système de forces, au moment t, a l'expression :

$$R_1(t) = F_e(t) = F_r(t) = -K_2 . \dot{x}_2 = -K_2(\dot{x} - \dot{x}_1) = -K_2 . \dot{x} + K_2 .$$

$$\left(-\frac{\dot{R}_1(t)}{K}\right) \Rightarrow \dot{R}_1(t) = \frac{K_1}{K_2} . R_2(t) = -K_1 . \dot{x}.$$

La solution générale de cette équation différentielle a la forme :

$$R_1(t) = e^{-K_1 . tK_2}\left[C + K_1\int_0^t\left(e^{K_1 . \tau K_2} . \frac{dx}{d\tau}\right) . d\tau\right]$$

En supposant que $R_1(t)|_{t=0}^{=r_1}$, on obtient :

$$R_1(t) = e^{-K_1 . t/K_2} . \left[r_1 + K_1\int_0^t\left(e^{K_1 . \tau/K_2} . \frac{dx}{d\tau}\right) . d\tau\right]$$

Ainsi nous obtenons la résultante de ce système des forces, au moment t, appliquée au corps de masse m :

$$R(t) = -K_1 \cdot e^{-K_1 t/K_2} \cdot \int_0^t \left(e^{K_1 \tau/K_2} \cdot \frac{dx}{d\tau} \right) d\tau - r_1 \cdot e^{-K_1 t/K_2} + q(t).$$

En effectuant la projection de la loi fondamentale de la dynamique sur l'axe x on obtient l'équation intégro-différentielle :

$$\text{(I)} \qquad m \cdot \frac{d^2x}{dt^2} + K_1 \int_0^t \left(e^{K_1(\tau - t)/K_2} \cdot \frac{dx}{d\tau} \right) \cdot d\tau = F(t), \; t \geqslant 0, \text{ oú}$$

$$F(t) = q(t) - r_1 \cdot e^{-K_1 t/K_2}$$

L'équation (I) représente l'équation du mouvement du modèle Maxwell. Nous nous proposons d'intégrer cette équation en cherchart la solution qui doit vérifier les conditions initiales :

$$\text{(II)} \qquad x(t) \Big|_{t=0^+} = x_0 \qquad \dot{x}(t) \Big|_{t=0^+} = v_0 .$$

Le noyau de l'équation (I) est $K(t) = e^{-K_1 t/K_2}$.

Maintenant introduisons les fonctions : $\overline{X}(t) = \theta(t) \cdot x(t)$, $\overline{K}(t) = \theta(t) K(t)$ $\overline{F}(t) = \theta(t) \cdot F(t)$, auxquelles nous attachons les distributions régulières correspondantes, qui sont notées aussi par $\overline{X}, \overline{K}, \overline{F}$. L'équation (I) s'écrit :

$$\text{(III)} \qquad m \frac{\tilde{d}^2 \overline{X}}{dt^2} + K_1 \cdot K(t) \cdot \frac{\tilde{d}\overline{X}}{dt} = \overline{F}(t).$$

Les opérations de dérivation dans l'équation (III) ont le sens classique. Le signe „~" indique les distributions régulières attachées aux mêmes dérivées au sens commun. En utilisant les formules de liaison entre les dérivées au sens commun, des fonctions discontinues, et des dérivées au sens de la théorie des distributions et en même temps les propriétés du produit de convolution, l'équation (III) devient :

$$\text{(IV)} \qquad [m\delta''(t) + K_1 \cdot \overline{K}(t)] \ast \overline{X}(t) = \overline{F}(t) + m v_0 \delta(t) + m x_0 \delta'(t) - K_1 \cdot x_0 \overline{K}(t),$$

L'équation (IV) corespond, dans les distributions, à l'équation intégro-différentielle (I) et aux conditions initiales (II). La solution générale de l'équation (IV) est representée par l'égalité :

$$\text{(V)} \qquad \overline{X}(t) = \overline{F}(t)[\overline{F}(t) + m v_0 \delta(t) + m x_0 \delta'(t) - K_1 x_0 \cdot \overline{K}(t)]$$

où $\overline{F}(t)$ représente la solution fondamentale de l'équation (IV). La solution fondamentale $\overline{F}(t)$ vérifie l'équation :

$$\text{(VI)} \qquad [m\delta''(t) + K_1 \cdot \overline{K}(t)] \ast \overline{F}(t) = \delta(t).$$

En appliquant la transformée de Laplace on obtient :

$$\text{(VII)} \qquad L[F(t)_n] = \frac{1}{m} \cdot \frac{2 \cdot \alpha \cdot p + \omega^2}{P(2\alpha p^2 + \omega^2 \cdot p + 2\alpha\omega^2)}, \text{ où } 2\alpha = K_2/m;$$

$\omega^2 = K_1/m$ et dans ce cas nous sommes conduit aux cas :

A. Si $\omega > 4\alpha \Rightarrow \overline{F}(t) = (A_1/2m\alpha + A_2 \cdot e^{\lambda_1 t}/m + A_3 \cdot e^{\lambda_2 t}/m) \cdot Q(t)$ où

$$A_1 = 1; \; A_2 = \frac{2\alpha\lambda_1 + \omega^2}{2\alpha\lambda_1(\lambda_1 - \lambda_2)}; \; A_3 = \frac{2\alpha\lambda_2 + \omega^2}{2\alpha\lambda_2(\lambda_2 - \lambda_1)};$$

λ_1 et λ_2 sont les deux racines réels, négatives du trinôme $2\alpha p^2 + \omega^2 \cdot p + 2\alpha\omega^2$

$$\text{B. Si } \omega = 4\alpha \Rightarrow \overline{F}(t) = \left[\frac{2}{m\omega} - \frac{1}{m}\left(\frac{2}{\omega} + t\right) \cdot e^{-\omega t}\right] \theta(t)$$

$$\text{C. Si } \omega < 4\alpha \Rightarrow \overline{F}(t) = \left[\frac{1}{2m\alpha} + \frac{3\omega^2 - 8\alpha^2}{2m\alpha\omega\sqrt{16\alpha^2 - \omega^2}} \cdot e^{-\frac{\omega^2}{4\alpha}t} \cdot \right.$$

$$\left. \cdot \sin\frac{\omega}{4\alpha} \cdot \sqrt{16\alpha^2 - \omega^2}\, t - \frac{1}{2m\alpha} \cdot e^{-\frac{\omega^2}{4\lambda}} \cdot \quad \cdot \cos\frac{\omega}{4\alpha} \cdot \sqrt{16\alpha^2 - \omega^2\, t}\right] \cdot \theta(t).$$

Pour la force perturbatrice $F(t)$ nous pouvons considérer des différents cas particuliers : choc au moment initiale, choc au moment initial et puis maintenu constant, une excitation harmonique etc.

STUDIUL ECUAŢIILOR INTEGRO-DIFERENŢIALE ÎN SPAŢIUL DISTRIBUŢIILOR. APLICAŢIE : STUDIUL ECUAŢIEI DE MIŞCARE A MODELULUI MAXWELL

(Rezumat)

Se consideră ecuaţia integro-diferenţială de forma :

$$P\left(\frac{d}{dx}\right) \cdot y(x) + \lambda \int_0^x K(x, t) \cdot Q\left(\frac{d}{dt}\right) \cdot y(t) \cdot dt = f(x), \quad x \geqslant 0$$

şi condiţiile iniţiale : $y(0) = y_0^i$, $i = 0, 1, 2, \ldots, n-1$.

Corespondentul în teoria distribuţiilor a problemei Cauchy anterioare este ecuaţia de convoluţie în $D'_+ + (R)$:

$$\left[P\left(\frac{d}{dx}\right)\delta(x) + Q\left(\frac{d}{dx}\right) \cdot \overline{K}(x)\right] \ast \overline{y}(x) = \overline{f}(x) + \sum_{k=0}^{n-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{n-1} a_i \cdot y_0^j\right) \cdot$$

$$\cdot \delta_{(x)}^{(n-k-1)} + \sum_{k=0}^{m-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{m-1} b_i y_0^j\right) \overline{K}_{(x)}^{(m-k-1)} = b(x)$$

Soluția acestei ecuații va fi:
$\overline{y}(x)=\overline{F}(x)*b(x)$ unde $\overline{F}(x)$ este soluția fundamentală ce se obține aplicînd transformata Laplace inversă ecuației:

$$\perp[F(x)]=\frac{1}{\mathrm{L}\left[P\left(\frac{d}{dx}\right)\delta(x)+Q\left(\frac{d}{dx}\right)\overline{K}(x)\right]}.$$

Ecuația de mișcare a modelului Maxwell este ecuația integro-diferențială:

$$m\frac{d^2x}{dt^2}+K_1\int_0^t\left(e^{K_1(\tau-1)/K_2}\cdot\frac{dx}{d\tau}\right)d\tau=F(t),\ t\geq 0.$$

Se consideră și condițiile inițiale:

$$x(t)\mid t=0+=x_0;\quad \dot{x}(t)\mid t=0+=v_0.$$

Corespondentul în teoria distribuțiilor a acestei probleme Cauchy este ecuația de convoluție în $D'_+(R)$:

$$[m\cdot\delta''(t)+K_1\cdot K,(t)]*\overline{x}(t)=\overline{F}(t)+mv_0\delta(t)+mx_0\delta'(t)-K_1x_0\overline{K}(t),$$

Conform teoriei generale, soluția fundamentală a acestei ecuații se obține din ecuația:

$$\perp L[F(t)]=\frac{1}{\mathrm{L}[m\delta''(t)+K_1\overline{K}(t)]}$$

iar soluția ecuației de convoluție va fi:

$$\overline{x}(t)=\overline{F}(t)+[\overline{F}(t)+mv_0\delta(t)+mx_0\delta(t)-K_1x_0\overline{K}(t)]$$

THE INTEGRO-DIFFERENTIAL EQUATION ANALYSIS IN THE DISTRIBUTION SPACE. THE ANALYSIS OF MAXWELL'S MOUVEMENT EQUATION.

(Abstract)

It is considered the following form of the integro-differential equation:

$$P\left(\frac{d}{dx}\right)\cdot y(x)+\lambda\cdot\int_0^x K(x,t)\cdot Q\left(\frac{d}{dx}\right)\cdot y(t)dt=f(x),\quad x\geq 0.$$

and the initial conditions:

$$y^{(i)}(0)=y_0^i,\quad i=0,1,2,\dots,n-1$$

This Cauchy problem correspond in the distribution theory to the convolution equation in the $D'_+(\mathrm{R})$:

$$\left[P\left(\frac{d}{dx}\right)\cdot\delta(x)+Q\left(\frac{d}{dx}\right)\cdot\overline{K}(x)\right]*\overline{y}(x)=\overline{f}(x)+\sum_{k=0}^{n-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{n-1}a_i\cdot y_0^j\right)\cdot$$

$$\cdot\,\delta_{(x)}^{(n-k-1)}+\sum_{k=0}^{m-1}\left(\sum_{\substack{i,j=0\\ i+j=k}}^{m-1}b_iy_0^j\right)\overline{K}_{(x)}^{(m-k-1)}=\overline{b}(x)$$

which has the solution:

$\overline{y}(x)=\overline{F}(x) \ast b(x)$ where $\overline{F}(x)$ is the basic solution given by Laplace transformation of the equation:

$$L\,[\overline{F}(x)] = \frac{1}{L\left[P\left(\frac{d}{dx}\right)\delta(x) + Q\left(\frac{d}{dx}\right)\overline{K}(x)\right]}.$$

The Maxwell's model equation of the mouvement is given by the integro-differential equation:

$$m\frac{d^2x}{dt^2} + K_1\int_0^t \left(e^{K_1(\tau-t)/K_2}\frac{dx}{d\tau}\right)d\tau=\overline{F}(t),\ t \geqslant 0$$

with the initial conditions:

$$x(t)\,|\,t = 0^+ = x_0,\quad \dot{x}(t)\,|\,t = 0^+ = v_0$$

This Cauchy problem correspond in the distribution theory, to the convolution equation in the $D'_+(R)$:

$$[m\delta''(t) + K_1\overline{K}'(t)] \ast \overline{x}(t) = \overline{F}(t) + mv_0\delta(t) + mx_0\delta'(t) - K_1x_0\,\overline{K}(t)$$

Following the general theory it is possible to find out the basic solution from equation

$$L\,[\overline{F}(t)] = \frac{1}{L\,[m\delta''(t) + K_1\overline{K}'(t)]}$$

and the solution of the convolution equation is:

$$\overline{x}(t) = \overline{F}(t) + [\overline{F}(t) + mv_0\delta(t) + mL_0\delta'(t) - K_1x_0\overline{K}(t)].$$

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CONTRIBUTIONS TO THE STUDY OF STOCHASTIC DYNAMICS OF MACHINE-TOOLS

G. BĂLAN, V. POPA, I. STRAT

1. Introduction. In the study of dynamic behaviour of machine-tools different types of excitations are used :

a) Sinusoidal excitation [7] ; the disadvantage of this method is the long time it takes.

b) Excitation by impulse or unit function [7].

The general inconvenient of the above mentioned deterministic signals is their alteration by stochastic disturbances (noises), which actually exist during the working of machine-tools.

c) Stochastic excitation ; lately this type of stimulation has imposed itself, showing the following advantages :

— it eliminates the influence of disturbances ;

— they can be applied by superposing them on the current input value, the normal working of the machine is not interrupted.

In [11], the structure of the machine is stimulated by an electro-hydraulic shaker, directed by a noise generator, the input and output signals having — in time — a stochastic configuration.

A common shortcomming of the above mentioned papers is that the dynamic of the machine is not determined in actual conditions of cutting ; the omission is important, as the behaviour of the machine is influenced by the pivoting motion, the dimension of the piece, the part of the piece that is being cut. The dynamics of machine-tools during the cutting, applying a stochastic stimulation force, or by the cutting of a piece whose surface was streaked stochastically is studied in [4, 10].

The aim of this paper is :

— to propose a new method to achieve the stochastic excitation for machine-tools, during their work and

— to determine the statistic characteristics and to show the utility of the experimentally obtained signals.

2. Achieving the stochastic excitation. To obtain a stochastic variation of the cutting force, a cylinder of cast iron, with porosities obtained by adding a quantity of water in the casting form, was longitudinally shaped. The thrust force F_y was measured by the help of two strain gages — 1 and 2 — applied on the inferior, respectively on the superior surface of the cutting tool (fig. 1), conf. with [5].

A cutting tool armed by cemented-carbide tip was used, with the angles :

$$\varkappa=70^\circ ; \quad \varkappa_s=60^\circ ; \quad \lambda=5^\circ ; \quad \alpha=10^\circ ; \quad \gamma=15^\circ$$

and the dimensions : $h=38{,}5$ mm ; $a=19{,}5$ mm ; $b=12$ mm (thickness).

The strain gages are of type RFT, with the factor $K=\frac{\Delta R}{R}:\frac{\Delta I}{I}=1{,}41$; it was worked with short gages.

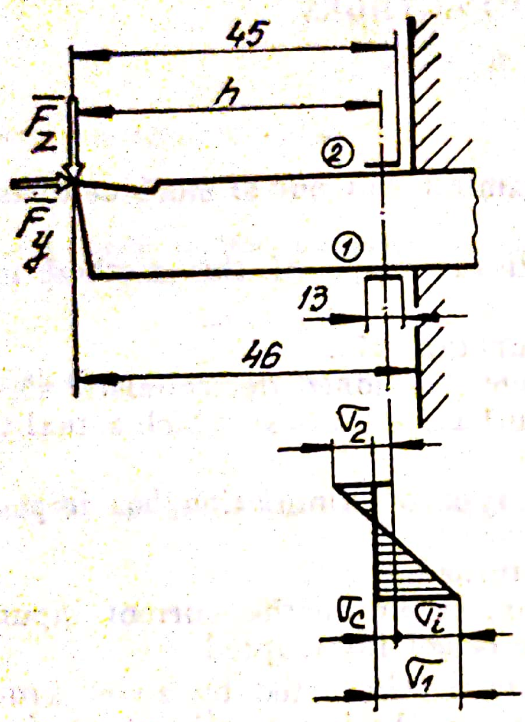

Fig. 1

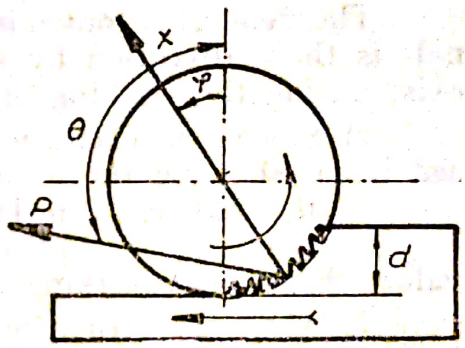

Fig. 2

The tool is required at the bending by the force F and at compression by F_y. To reduce the 3rd component of the cutting force F_x (in order not to influence the transversal sensitivity of the strain gages) it was worked with small length feed.

The two strain gages measure the relative deformations ε_1 and ε_2, by the help of which the unitary efforts indicated in fig. 1 can be calculated :

(1) $$\sigma_1=E\,.\,\varepsilon_1 ; \quad \sigma_2=E\,.\,\varepsilon_2$$

where $E=2{,}1\,.\,10^4$ $\mathrm{da}^N/\mathrm{mm}^2$

(2) The unitary compression efforts $\sigma_c=\dfrac{F_Y}{a\,.\,b}$

(3) and of bending $\sigma_i=\dfrac{M}{W}=\dfrac{F_z\,.\,h-F_y\,.\,a/2}{ba^2/6}$,

are connected to the relations :

(4) $$\sigma_1 = \sigma_i + \sigma_c \; ; \qquad \sigma_2 = \sigma_i - \sigma_c$$

from where:

(5) $$\sigma_i = \frac{\sigma_1 + \sigma_2}{2} \; ; \qquad \sigma = \frac{\sigma_1 - \sigma_2}{2} \, .$$

It results:

(6) $$F_y = \frac{ab}{2}(\sigma_1 - \sigma_2) \; ; \quad F_z = \frac{a^2 b}{6h}(2\sigma_1 - \sigma_2)$$

For the thermic compensation a tensometric connection identic with that from fig. 1, is made on another tool.

The gages are connected with a tensometric bridge TDA-3 (R.S. Czechoslovacia), and the outputs of the two bridge chanals are each connected to a spot of the oscillograph type Honeywell Automatic Control Visicorder 3508 – GMBHM. The experiment was done on Romanian lathe SNA-400 in a laboratory of the Department for Applied Mechanics and Naval Installations of Galatz University.

The relative moving tool piece on the thrust force direction was measured by a tastograph type Metallwerker K.–G., Meerane Sachs, 1., 1408/1719, mounted on a support connected to the tool block.

In the study of the dynamic behaviour of the lathe the thrust force F_y, which causes the relative deformation of the tool picee Δy, matters as this affects directly the dimensional precision and precision of cylindric form; the force F_z, though bigger than F_y, is not significant because the deformation Δ_z is transmitted only to a small extent and it is inversely proportional with the diametre of the piece [8].

Any machine-tool can be considered as a system for which the input is represented by certain components of the force or of the moment of cutting, and the output represents some component of the relative displacement tool piece.

For example – with the milling machine (fig. 2), the input is the total cutting force P, calculated as resultant of the components acting on each of the tooths in cutting, passing through the centre of the arc which defines the contact surface tool-piece, and the output is the displacement on the direction x – normal with the reshaped surface.

3. Registering the variations of the cutting force F_y. We proceeded as follows:

a) The zero line was marked on the oscillograph paper (the tool is not charged).

b) The standardisation of gages was done charging (statically) the tool – on the front face and near the top – with a known weight G. The strain gage 2 being required at stretching, and gage 1 at bending, the two spots deviate in contrary senses. The weight G causes a relative extension in the sticking zone of the gages:

(7) $$\varepsilon = \frac{\sigma}{E} = \frac{1}{E} \cdot \frac{M}{W} = \frac{1}{E} \cdot \frac{G \cdot h}{ba^2/6} = \frac{6h}{Eba^2} \cdot G = \frac{6 \cdot 3{,}85}{2{,}1 \cdot 10^6 \cdot 1{,}2 \cdot 1{,}95^2} \cdot G ,$$

$$\varepsilon = 2{,}41 \cdot 10^{-6} \cdot G$$

where G is measured in (daN).

Experiments were made with $G'=10,85$ daN and $G''=30$ daN, for which :

$$\varepsilon'_{real} = 26,2 \,.\, 10^{6} \tag{8}$$
$$\varepsilon''_{real} = 72,2 \,.\, 10^{6}.$$

In the case of bending of the tool, without axial compression, the relations (5) become :

$$\sigma_c = \frac{\sigma_1 - \sigma_2}{2} = 0 \; ; \quad \sigma_i = \frac{\sigma_1 + \sigma_2}{2} = \sigma,$$

from where :

$$\sigma_1 = \sigma_2 = \sigma = E \,.\, \varepsilon, \tag{9}$$

that is, the gages suffer symetric deformations, as it can be seen in fig. 3, copied after registering (short registrations were done and the displacements of the spots ε' and ε'', according to the charging with G' and G''). The asymetry in the registration (fig. 3) is considered by us to result from nonlinearities.

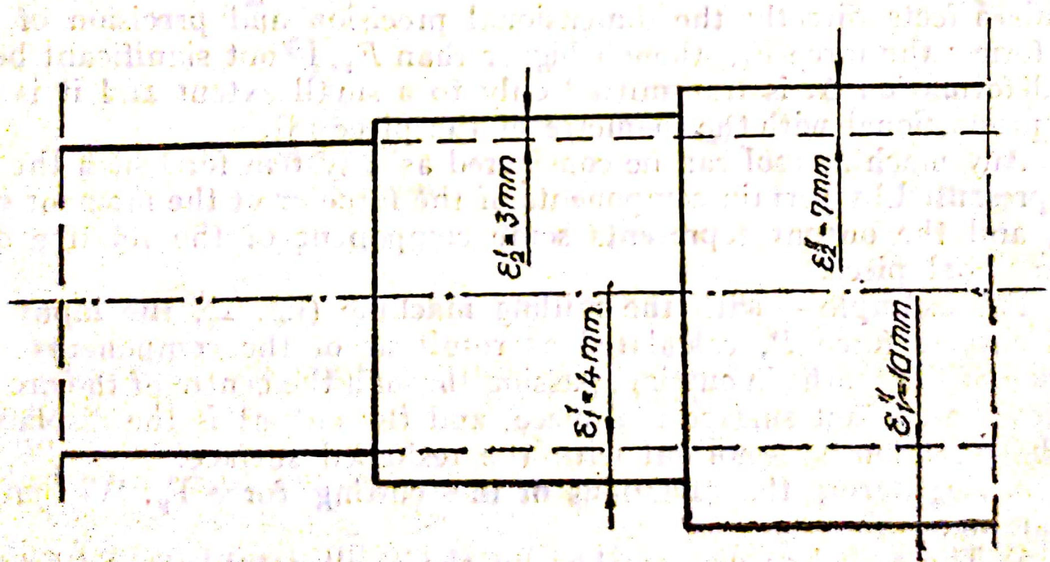

Fig. 3

In medium :

$$\varepsilon'_{registr.} = \frac{\varepsilon_1' + \varepsilon_2'}{2} = 3,5 \text{ mm} \; ; \quad \varepsilon''_{registr.} = \frac{\varepsilon_1'' + \varepsilon_2''}{2} = 8,5 \text{ mm} \tag{10}$$

The scale of the relative extention :

$$K' = \frac{\varepsilon'_{real}}{\varepsilon'_{registr.}} \; ; \quad K'' = \frac{\varepsilon''_{real}}{\varepsilon''_{registr.}}$$

were taken

$$K_F=\frac{1}{2}(K'+K'')=\frac{1}{2}\left(\frac{26{,}2\cdot 10^{-6}}{3{,}5}+\frac{72{,}2\cdot 10^{-6}}{8{,}5}\right)$$

(11)

$$K_F=\frac{\varepsilon_{real}}{\varepsilon_{registr.}(\mathrm{mm})}=8\cdot 10^{-6}\mathrm{mm}^{-1}$$

c) Registrations during the work were made ; the speed of the oscillograph paper : 50 mm/sec.

Recordings for four systems of cutting at longitudinal cutting, were made in accordance with tab. 1.

Tabel 1

Parameter \ Registration	I	II	III	IV
t (mm)	4	4	7	7
n (rot./min.)	250	500	250	500
s (mm/rot.)	0,2	0,2	0,2	0,2

For example, for the 3rd registration the curves looking like in fig. 4 were obtained.

At the beginning and at the end of registration, the zero line was marked, observing a deviation, owing (mainly) to termical noncompensation : the working tool warms, and the witness tool remains ar the temperature of the environment.

The qualitative aspect of the registration (fig. 4) we consider that satisfying, controlling the relation $\varepsilon_1>\varepsilon_2$, or $\sigma_1>\sigma_2$, resulting from (4) and fig. 1.

We make the assumption [9] that the zero derivation varies linearly in time ; therefore the initial and final zero points are joined with a straight line, and the ordinates of ε will be measured from this line.

The real values are obtained by the relation (11) :

(11′) $$\varepsilon_{real}=K_F\cdot\varepsilon_{registr.}=8\cdot 10^{-6}\cdot\varepsilon_{reg.}$$

The formulas (6) become :

(12) $$F_y=\frac{ab}{2}(\sigma_1-\sigma_2)=\frac{abE}{2}(\varepsilon_1^{real}-\varepsilon_2^{real})=\frac{abE\cdot K_F}{2}(\varepsilon_1^{reg.}-\varepsilon_2^{reg.})\,;$$

$$F_y=19{,}6(\varepsilon_1^{reg}-\varepsilon_2^{reg}).$$

(13) $$F_z=\frac{a^2b}{6h}(\sigma_1-\sigma_2)=\frac{a^2bE}{6h}(2\varepsilon_1^{real}-\varepsilon_2^{real})=\frac{a^2bEK_F}{6h}(2\varepsilon_1^{reg}-\varepsilon_2^{reg})$$

$$F_z=3{,}31(2\varepsilon_1^{reg}-\varepsilon_2^{reg})$$

In (12) and (13) the units of measure are : $[\varepsilon]=\mathrm{mm}$; $[F]=\mathrm{daN}$.

4. The Proccessing of the Registrations. To study the statistic characteristics of the force F_y, with stochastic variations in time, in order to etablish the utility of this signal in an identifying experiment on a lathe during working, an achievement of the stochastic process, obtained during the 3rd registering (tab. 1), was taken into consideration sufficiently long ($l=1$ m), sampled in 1 000 points — at a distance of 1 mm ; knowing the speed of the oscillograph paper ($v=50$ mm/sec.), there sesults the period of sampling :

$$t = 0{,}02 \text{ sec.} \tag{14}$$

The (absolute) ordinates of the curves ε_1^{reg} and ε_2^{reg} (fig. 4) were measured against the straight lines of the respective null drift and with the relation (12) the values of the force F_y, sampled by the step given by (14) were calculated.

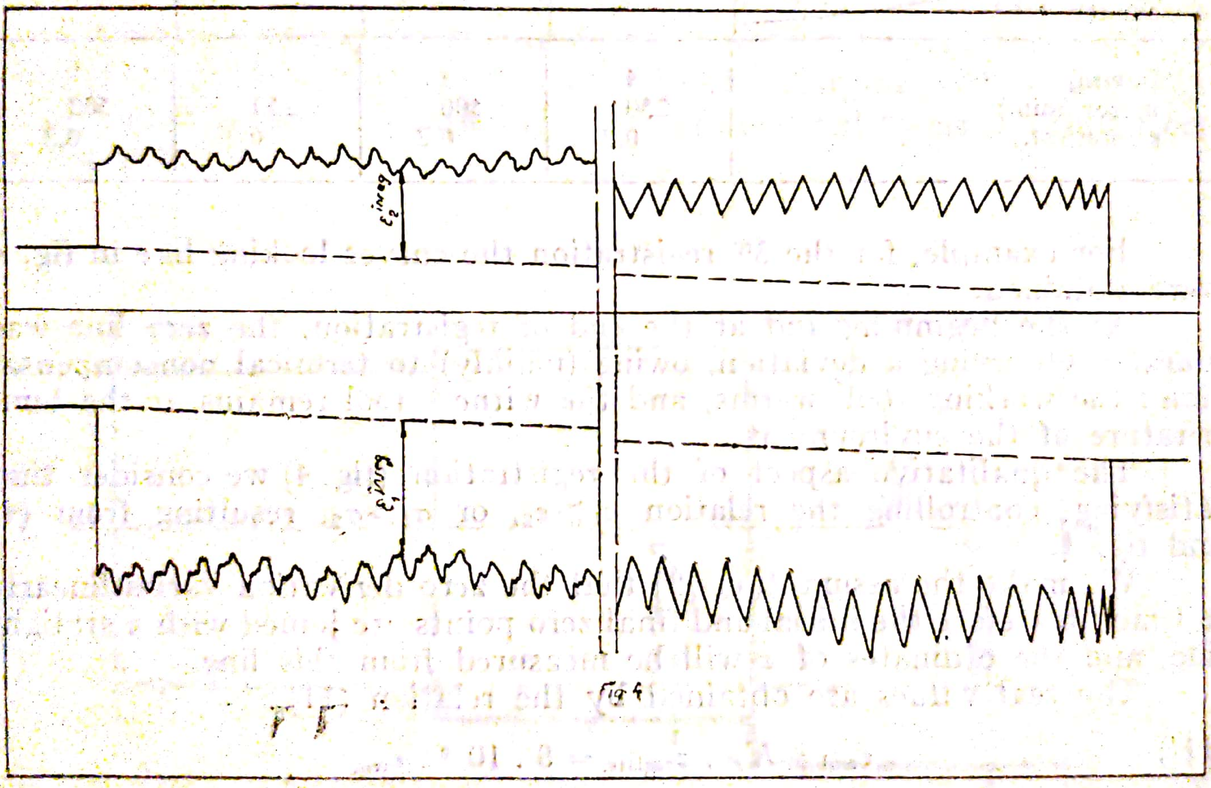

Fig. 4

Having the registration of the duration $T=(1\,000-1)\Delta t \cong 20$ sec., the approximate autocorrelation function is :

$$R_{ff}(\tau) = \frac{1}{T-\tau}\int_0^{T-\tau} F(t) \, . \, F(t+\tau) \, . \, dt. \tag{15}$$

The time was taken $T-\tau$ and not T, as in the latter case there would have been necessary registrations for the period $T+\tau$. For the actual calcu!ation, the integral (15) becames the sum :

(16)
$$R_{ff}(K \cdot \Delta) \simeq \frac{1}{N-K-1} \sum_{m=0}^{N-K-1} F(m\Delta) \cdot F[(m+K)\Delta] = \frac{1}{N-K-1} \sum_{m=1}^{N-K} F(m\Delta) \cdot F[(m+K) \cdot \Delta],$$

where :

N — is the total number of discret values — $N=1\,000$;
Δ — is the sampling interval of the stochastic function F, given by (14) ;
K — the discret delay parameter ($K \cdot \Delta = \tau$) ;
m — the discret current variable ($m \cdot \Delta = t$) ;
F — the centring value of the stochastic variable F_y :

(17)
$$F = F_y - \overline{F}_y,$$

where :

(18)
$$\overline{F}_y = \frac{1}{N} \sum_{i=1}^{N} F_{yi}$$

In case we work with centring values of the stochastic variable, the autocorrelation function becomes an autocovariant function.

The maximum value of the autocorrelation function, achieved for $\tau=0$, is equal with the root—mean—square value of the stochastic process and, in case of the centring stochastic process, it is equal with the dispersion, too :

(19)
$$R_{ff}(0) = \frac{1}{T} \int_0^T F^2(t) \cdot dt = m^2 = D$$

On the other hand, dispersion can be calculated by transforming the integral form (19) in the sum :

(20)
$$D \simeq \frac{1}{N-1} \sum_{i=1}^{N} F_i^2$$

The spectral density function represents (according to the Hincin-Wiener theorem) the direct Fourier transformation of the autocorrelation function :

(21)
$$S_{ff}(\omega) = \int_{-\infty}^{\infty} R_{ff}(\tau) \cdot e^{-i\omega\tau} \cdot d\tau = \int_{-\infty}^{\infty} R_{ff}(\tau) \cdot \cos \omega\tau \cdot d\tau - i \int_{-\infty}^{\infty} R_{ff}(\tau) \cdot \sin \omega\tau . d\tau = 2\int_0^{\infty} R_{ff}(\tau) \cdot \cos \omega\tau \cdot d\tau$$

In the relations (21) the fact that the autocorrelation function is an even function, not negative for a stationar process was taken into

account; it results that the spectral density function is a real function, even and positive.

(22) It is approximated : $S_{ff}(\omega) \simeq 2\int_0^T R_{ff}(\tau) \, . \cos \omega\tau \, . \, d\tau,$

wehre T is the time after which $R_{ff}(\tau) \simeq 0$, and the integral is transformed in the sum ;

(23) $$S_{ff}(K \, . \, \Delta\omega) \simeq 2 \, . \, \Delta t \int_{i=0}^{N-1} R_{ff}(i \, . \, \Delta\tau) \, . \cos (i \, . \, K \, . \, \Delta\tau \, . \, \Delta\omega)$$

The relations (12, 18, 17, 20, 15 and 23) were estimated by the computer Felix C 32, having in view that :

— the autocovariant function was calculated varying K with a step of 4 units, so the sampling step is

(24) $$\Delta\tau = 4 \, . \, \Delta t = 0{,}08 \text{ sec.} ;$$

— for the spectral density function was chosen

(25) $$\Delta\tau \, . \, \Delta\omega = \frac{\pi}{90}$$

resulting that the „cosinus" function from (23) is calculated only for even an whole number of grades ;

— the sampling period for $S_{ff}(\omega)$:

$$\Delta\omega = \frac{\pi}{90 : \Delta\tau} = \frac{\pi}{7{,}2} s^{-1}$$

We remark that an analogue programme, but only for calculating the autocovariant function, is given in [3].

A part of the results printed in the programme written in FORTRAN, is given in chart 2.

Chart 2

i	1	101	201	301	401	501	601	701	801	901
F_{yi}	294	294	235,2	274,4	196	156,8	117,6	196	196	431,2

MEDIUM VALUE : $\bar{F}_y = 229{,}1$ daN
DISPERSION of F_y : D = 6 246

K	0	12	24	36	48	60	72	84	96	108	120	132
R $(K \Delta)$	6 324	1 090	3 861	2 099	2 237	2 596	1 707	2 402	2 101	1 271	2 416	662,7

144	156	168	180	192	204	216	228	240	252	264	276	280
1 869	658,4	1 232	1 220	388,8	639.1	364,7	107	328,2	– 15,97	327	– 306	22,56

K	0	8	16	24	32	40	48	56	64	72
$S_{ff}(K\Delta\omega)$	16 091	1 123,5	559,2	546,6	550,7	571,2	586,3	592,8	604	594,7

80	88	96	104	112	120	128	136	144
887	1 740	3 793,6	838,5	602	520	501	510	539

It can be observed that the relation (19) is verified.

The autocovariant $R_{ff}(\tau)$ and spectral density $S_{ff}(\omega)$ functions are graphically represented in figures 5 and 6.

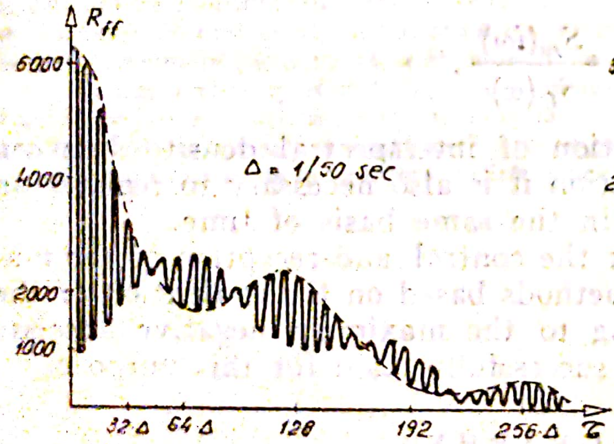

Fig. 5

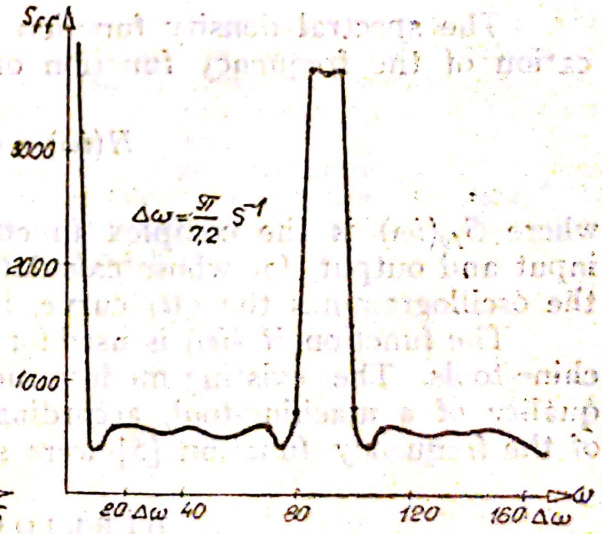

Fig. 6

Conclusions. As seen in fig. 5, the autocovariant function tends towards zero after a sufficiently long time, this being the sufficient condition of ergodicity [3]; the processing of only one achievement of the stochastic process is thus justified.

The function $R_{ff}(\tau)$ decreases rapidly, a fact that indicates a rapid variation in time of the stochastic process, which in this way contains o components of high frequency (there is a high intersection frequency of the time axis). If the stochastic function varies slowly in time, a much

greater relative displacement is necessary in order to correspond to the ordinates of opposite signs ; in this case the corelograma has not any longer a pronounced maximum. If the sampling step had been $\Delta\tau'=8\,.\,\Delta t$, the autocovariant function would have had the graphical representation with a broken line.

The spectral density function is an even function, non negative in pulsation and described in what way the power of the stochastic signal is distributed on the pulsations. The flat graphic of the function denotes the presence of a great number of higher frequencies (pulsations).

The presence of the 2nd maximum of the function $S_{ff}(\omega)$, situated at the pulsation :

$$\omega^* \simeq 90\,.\,\Delta\omega = 90\,.\,\frac{\pi}{7,2} = 39,4\ s^{-1}$$

can be explained by the presence in the autocovariant function of a periodic process — noticcable in fig. 5 — of the period.

$$T^* \simeq 8\,.\,\Delta = 8\,.\,\frac{1}{50} = 0,16 \text{ sec},$$

to which corresponds the pulsation

$$\omega^* = \frac{2\pi}{T^*} = \frac{2}{0,16} = 39,2\ s^{-1}$$

The spectral density function $S_{ff}(\omega)$ is necessary for the identification of the frequency function of the machine :

$$H(i\omega) = \frac{S_{fy}(i\omega)}{S_{ff}(\omega)},$$

where $S_{fy}(i\omega)$ is the complex function of interspectral density between input and output, for whose calculation it is also necessary to register on the oscillogrammes the $y(t)$ curve, in the same basis of time.

The function $H\ (i\omega)$ is used for the control and reception of the machine-tools. The existing modern methods based on the evaluation of the quality of a machine-tool, according to the maximum negative abscisse of the frequency function [5] were successfully used for this purpose.

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CONTRIBUŢII LA STUDIUL DINAMICII STOCASTICE A MAŞINILOR-UNELTE

(Rezumat)

În lucrare se propune o nouă metodă de realizare a excitaţiei aleatoare la maşini-unelte (în timpul funcţionării lor); se determină caracteristicile statistice şi se arată utilitatea semnalului obţinut experimental. Excitaţia aleatoare, reprezentată la strung de variaţia în timp a forţei de aşchiere de respingere Fy, , s-a măsurat tensometric şi înregistrat pe un oscilograf.

Caracteristicile statistice ale procesului aleator sînt reprezentate de funcţiile de autocorelaţie şi de densitate spectrală şi s-au calculat pe un calculator Felix C 32. Din studiul acestor caracteristici s-a evidenţiat ergodicitatea procesului şi utilitatea lui în identificarea funcţiei de răspuns la frecvenţă a maşinii.

DES CONTRIBUTIONS POUR L'ETUDE DE LA DYNAMIQUE STOCHASTIQUE DES MACHINES—OUTILS

(Resumé)

L'ouvrage présente une nouvelle méthode pour obtenir une excitation aléatoire aux machines-outils (pendant leur fonctionnement); on détermine les caractéristiques statistiques et on montre l'outilité du signal obtenu expérimentalement. L'excitation stochastique représentée pour un tour — par la variation en temps de la force de coupe de repulsion F_y, a étè mesuré tensométriquement et a été enregistrée par un oscillograph.

Les caractéristiques statistiques du processus aléatoire sont représentées par les fonctions d'autocorrelation et de densité spectrale et elles ont été calculées pur un computer Felix C 32. Par l'étude de ces caractéristiques a été mise en evidence l'érgodicité du processus et son outilité pour l'identification de la fonction de fréquence de la machine.

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ABOUT THE LAW OF UNIVERSAL GRAVITY[1]

BY

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In this paper we shal leave aside the heliocentrical outlook — which was drawn out, for the first time, by A r i s t a r h from S a m o s (310—230 a.H) and rediscovered by *C o p e r n i c* 1800 years later — and deal with the vegacentrical outlook, outgoing from the basic idea that on every particle or free body, moving in the gravitational field, acts a force whose module is dependent on a side deviating force, proper to Universe structure.

This idea will be put at the base of the study of the solar system motion towards Vega star (apex direction) and the equation obtained, (1), will be used for comparison with the experimental data.

For evidencing this motion, we shall assume a coordinate system, linked to Vega star, but not participating to the rotation motion of this star, fig. 1. We shall also neglect the curvature of the trajectory described by the Sun in his translation motion towards Vega star.

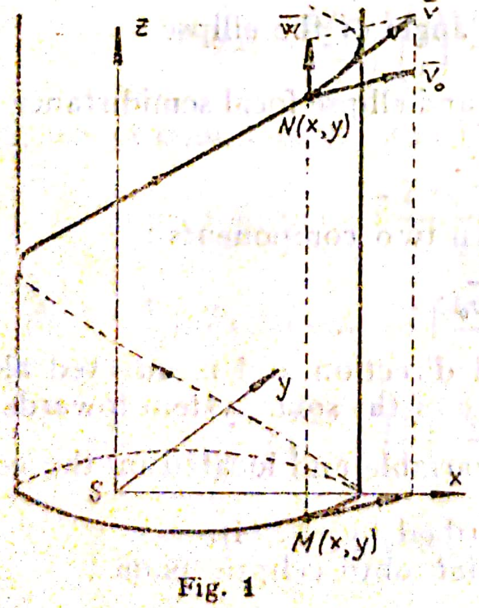

Fig. 1

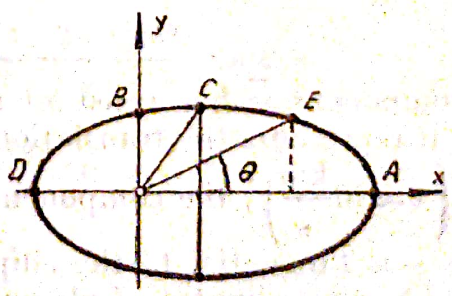

Fig. 2

[1]) This paper has been presented to the public in Romania and submitted for publishing in „Nature", London, in September 1982.
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The firm plane of coordinates xy will be the plane of the ecliptic, with the Sun in the focus S of the ellipse, with the axis Sx directed to aphelion and the axis Sy perpendicular to x — axis. The Sz axis, normal to xy plane, represents the Sun motion direction, fig. 2.

The equation describing the motion in the right — handed system, $(\bar{i}\times\bar{j}=\bar{k})$, fig. 1, shall be:

$$m\dot{\bar{v}}=\bar{F}+[(Cm\bar{v}\times \text{rot } \bar{v})\bar{u}_0]\bar{u}_0 \tag{1}$$

where:

m is the mass of the considered planet,

$\bar{v}$ — the planet velocity, tangent to helical trajectory of the planet,

rot $\bar{v}$ — the planet velocity curl,

C — a non-dimensional universal constant,

$\bar{u}_0$ — the unit vector of the force $\bar{F}$,

$$|\bar{F}|=K\frac{M_s m}{r^2}, \quad K=6{,}664.10^{-11}\left[\frac{m^3}{\text{kg s}^2}\right] \tag{2}$$

The side deviating force, given by the interaction between free body mass and its rotorical fields, represents the vectorial component of the gravitic field; it is:

$$\bar{F}_d=Cm\bar{v}\times \text{rot } \bar{v} \tag{3}$$

The planet trajectory projected on ecliptic plane is an ellipse of equation:

$$r(\theta)=\frac{p}{1-e\cos\theta}, \tag{4}$$

where:

r, θ are the vector radius and the polar angle of the ellipse

e — ellipse eccentricity

a, b, c — major semiaxis, minor semiaxis, and ellipse focal semidistance

$p=\frac{b^2}{a}=a(1-e^2)$ — an ellipse parameter.

The planet speed $\bar{v}$, may decomposed in two components:

$$\bar{v}=\bar{w}+\bar{v}_0; \tag{5}$$

the component $\bar{w}$ has constant size and direction and is directed along z axis; it represents the translation velocity of the solar system towards Vega star, $\left(w\approx 20\frac{\text{km}}{\text{s}}\right)$; the component $\bar{v}_0$ — variable and located in the ecliptic plane — is tangential to the ellipse described by eq. (4).

Let be the coordinates of planet projection onto ecliptic plane:

$$\begin{aligned} x&=r(\theta)\cos\theta \\ y&=r(\theta)\sin\theta \end{aligned} \tag{6}$$

It follows that

$$(\bar{v}_0)_x = \dot{x} = \left[\frac{\partial r}{\partial \theta}\cos\theta - r(\theta)\sin\theta\right]\dot{\theta} =$$

$$= \left[-\frac{e}{p}\,\frac{p^2}{(1-e\cos\theta)^2}\,\frac{\sin 2\theta}{2} - \frac{p}{1-e\cos\theta}\sin\theta\right]\frac{2\pi}{T}\left(\frac{a}{b}\right)^3(1-e\cos\theta)^2 \tag{7}$$

$$(\bar{v}_0)_y = \dot{y} = \left[\frac{\partial r}{\partial \theta}\sin\theta + r(\theta)\cos\theta\right]\dot{\theta} =$$

$$= \left[-\frac{e}{p}\,\frac{p^2}{(1-e\cos\theta)^2}\sin^2\theta + \frac{p}{1-e\cos\theta}\cos\theta\right]\frac{2\pi}{T}\left(\frac{a}{b}\right)^3(1-e\cos\theta)^2$$

where, on the basis of the theorem of surfaces

$$\dot{\theta} = \frac{2\pi}{T}\left(\frac{a}{b}\right)^3(1-e\cos\theta)^2 \tag{8}$$

According to equations (6), the equations (7) become:

$$\dot{x} = -\frac{2\pi}{T}\cdot\frac{a^2}{b}\cdot\frac{y}{r}$$

$$\dot{y} = \left(-\frac{b^2c}{a^2} + \frac{b^2x}{ar}\right)\cdot\frac{2\pi}{T}\cdot\left(\frac{a}{b}\right)^3, \tag{9}$$

which derived lead to:

$$\ddot{x} = -\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c\cos\theta)^2}{a}\cos\theta$$

$$\ddot{y} = -\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c\cos\theta)^2}{a}\sin\theta \tag{10}$$

Making use of equations (9) and (10) we may write:

$$\bar{v}_0 = \bar{i}\,\dot{x} + \bar{j}\,\dot{y} = -\bar{i}\,\frac{2\pi}{T}\,\frac{a^2}{b}\,\frac{y}{r} + \bar{j}\left(-\frac{b^2c}{a^2} + \frac{b^2x}{ar}\right)\frac{2\pi}{T}\left(\frac{a}{b}\right)^3 \tag{11}$$

$$\dot{\bar{v}}_0 = \bar{i}\,\ddot{x} + \bar{j}\,\ddot{y} = -\bar{i}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c\cos\theta)^2}{a}\cos\theta -$$

$$-\bar{j}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c\cos\theta)^2}{a}\sin\theta \tag{12}$$

$$\operatorname{rot}\bar{v}_0 = \bar{k}\,\frac{2\pi}{T}\cdot\frac{a^2}{br} \tag{13}$$

As the vector of rot $\bar{v}_0$ has a constant direction, given by z — axis; if we write the vector product of the equation (5) with rot $\bar{v}_0$, we shall obtain:

$$\bar{v}\times\operatorname{rot}\bar{v}_0 = \bar{w}\times\operatorname{rot}\bar{v}_0 + \bar{v}_0\times\operatorname{rot}\bar{v}_0 = \bar{v}_0\times\operatorname{rot}\bar{v}_0; \tag{14}$$

on other hand, we have :

$$\dot{\bar{v}} = \dot{\bar{v}}_0 \quad (\bar{w} = \text{ct.}), \tag{15}$$

hence, in the ecliptic plane the vectors $\bar{v}$ and $\dot{\bar{v}}$ have, respectively, the same projections as the vectors $\bar{v}_0$ and $\dot{v}_0$, given by eq. (9) and (11). Therefore, we may write :

$$\bar{v} \times \text{rot}\, \bar{v} = \bar{i}\left(\frac{2\pi}{T}\right)^2 \left[-\left(\frac{a}{b}\right)^3 \frac{bc}{r} + \left(\frac{a}{b}\right)^2 \frac{a^2}{r^2}\, x\right] + \bar{j}\left(\frac{2\pi}{T}\right)^2 \left(\frac{a}{b}\right)^2 \frac{a^2}{r^2}\, y \tag{16}$$

Let us prove now that in every point of the trajectories of solar system planets, the constant C of the equation (1) maintains its constant value, equal to 0,007.131.089.24.

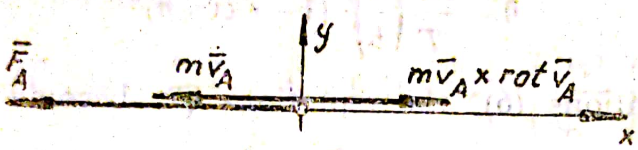

Fig. 3

Assuming the numerical values of the table (1) — which were used for universal constant C determination — and refering firstly to our planet we shall calculate in points A, B, C, E, D, fig. 2, the terms of equation (1) and taking into account their graphical plotting in oxy coordinate system, fig. 3, 4, 5, 6, 7, we shall prove that the formula :

$$C = \frac{|\bar{F} - m\dot{\bar{v}}|}{|[(m\bar{v} X \text{ rot}\, \bar{v})\bar{u}_0]\bar{u}_0|} \tag{17}$$

maintains its value in all the above stated points.

In the point A we have :

$$\theta = 0, \; x = r = a + c, \; y = 0 \tag{18}$$

and making use of eq. (12, (15), (16) and (2), we may write the following relationships in SI system :

$$m\dot{\bar{v}}_A = -\bar{i} m \left(\frac{2\pi}{T}\right)^2 \cdot \left(\frac{a}{b}\right)^4 \cdot \frac{(a-c)^2}{a} = -m\bar{v}_A \times \text{rot}\, \bar{v}_A =$$

$$= -\bar{i}\, 5{,}975.10^{24} \cdot 3{,}942.410.189.64 \cdot 10^{-14} \cdot 1{,}000.560.556.99 \cdot \tag{19}$$

$$\cdot \frac{147.001{,}602.048^2 \cdot 10^{12}}{149.504 \cdot 10^6} = -\bar{i}\, 3{,}406.704.153.41 \cdot 10^{22} [N]$$

$$F_A = K \frac{Mm}{(a+c)^2} = 6{,}664 \cdot 10^{-11} \cdot \frac{1{,}991 \cdot 10^{30} \cdot 5{,}975 \cdot 10^{24}}{152.006{,}397.952^2 \cdot 10^{12}} = \tag{20}$$

$$= 3{,}430.997.664.75 \cdot 10^{22} [N]$$

In view of the vectorial diagram of the forces in point A, fig. 3, the equation (17) gives:

$$C_A = \frac{|-3{,}430.997.664.75 + 3{,}406.704.153.41|}{|3{,}406.704.153.41|} = 0{,}007.131.089.24 \quad (21)$$

In the point B we have:

$$\theta = 90^\circ, \; x = 0, \; y = r = \frac{b^2}{a} \quad (22)$$

and hence we write the following relationships:

$$m\bar{v}_B = -\bar{j}m\left(\frac{2\pi}{T}\right)^2 \cdot \left(\frac{a}{b}\right)^4 \cdot a = -\bar{j}\,5{,}975 \cdot 10^{24} \cdot 3{,}942.410.189.64 \cdot$$

$$\cdot 10^{-14} \cdot 1{,}000.560.556.99 \cdot 149.504 \cdot 10^6 = -\bar{j}\,3{,}523.675.519.95 \cdot 10^{22}[N] \quad (23)$$

$$m\bar{v}_B x \operatorname{rot} \bar{v}_B = -\bar{i}m\left(\frac{2\pi}{T}\right)^2 \cdot \left(\frac{a}{b}\right)^4 \cdot c + \bar{j}m\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4 a = -\bar{i}\,5{,}975 \cdot 10^{24} \cdot$$

$$\cdot 3{,}942.410.189.64 \cdot 10^{-14} \cdot 1{,}000.560.556.99 \cdot 2502{,}397.952 \cdot 10^6 +$$

$$+ \bar{J}\,5{,}975 \cdot 10^{24} \cdot 3{,}942.410.189.64 \cdot 10^{-14}\, 1{,}000.560.556.99 \cdot \quad (24)$$

$$149{,}504 \cdot 10^6 = -\bar{i}\,0{,}058.979.280.852.7 \,.\, 10^{22}\,[N] + \bar{j}3{,}523.675.519.93.10^{22}[N]$$

$$F_B = K\frac{Mm}{\left(\frac{b^2}{a}\right)^2} = 6{,}664 \cdot 10^{-11} \cdot \frac{1{,}991 \cdot 10^{30} \cdot 5{,}975 \cdot 10^{24}}{\left(\frac{149.483.055{,}958}{149.504.000}\right)^2 149.483{,}055.958^2 \cdot 10^{12}} =$$

$$= 3{,}548.803.165.16 \cdot 10^{22}\ [N] \quad (25)$$

According to diagram of forces in fig. 4, we have:

$$C_B = \frac{|\bar{F}_B - m\bar{v}_B|}{|m\bar{v}_B \times \operatorname{rot} \bar{v}_B| \cos \beta_B} =$$

$$= \frac{|-3{,}548.803.165.16 + 3{,}523.675.519.95| \cdot 10^{22}}{|3{,}523.675.519.95| \cdot 10^{22}} =$$

$$= 0{,}007.131.089.41 \quad (26)$$

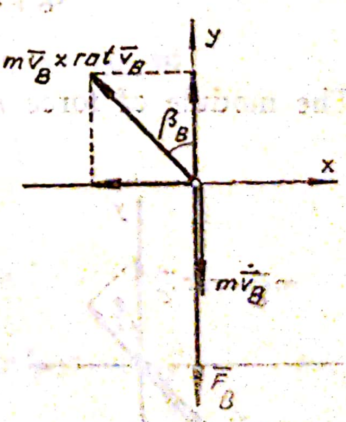

Fig. 4

In the point C we have

$$r = a, \; y = b = a \sin \theta, \; c = a \cos \theta \quad (27)$$

and using eq. (12), (15), (16) and (2) we may write:

$$m\bar{v}_C = -\bar{i}m\left(\frac{2\pi}{T}\right)^2 c - \bar{j}m\left(\frac{2\pi}{T}\right)^2 b =$$

$$= -\bar{i}\,5{,}975 \cdot 10^{24} \cdot 3{,}942.410.189.64 \cdot 10^{-14} \cdot 2502{,}397.952 \cdot 10^6 -$$

$$-\bar{j}\,5{,}975\cdot10^{24}\cdot3.942.410.189.64\cdot10^{-14}\cdot149.483{,}055.958\cdot10^{6}= \qquad (28)$$

$$=-\bar{i}\,58.946{,}238.127.1\cdot10^{16}-\bar{j}\,3{,}521.208.049.83\cdot10^{22}$$

$$m\bar{v}_C\times\mathrm{rot}\,\bar{v}_C=\bar{j}m\left(\frac{2\pi}{T}\right)^2\frac{a^2}{b}=$$

$$=\bar{j}\,5{,}975\cdot10^{24}\cdot3{,}942.410.189.64\cdot10^{-14}\cdot\frac{149.504^2\cdot10^{12}}{149.483{,}055.958\cdot10^{6}}= \qquad (29)$$

$$=\bar{j}\,3{,}522.194.830.49\cdot10^{22}\ [N]$$

While :

$$F_C=K\frac{Mm}{a^2}=6{,}664.10^{-11}\cdot\frac{1{,}991\cdot10^{30}\cdot5{,}975\cdot10^{24}}{149.504^2\cdot10^{12}}=$$

$$=3{,}546.814.973.09\cdot10^{22}\ [N], \qquad (30)$$

the components of this force along x and y axis are :

$$F_{C_x}=F_C\cos\theta=-F_C\cdot\frac{c}{a}=-3{,}546.814.973.09\cdot10^{22}\cdot\frac{2502{,}397.952\cdot10^{6}}{149.504\cdot10^{6}}=$$

$$=-0{,}059.366.589.01\cdot10^{22}\ [N] \qquad (31)$$

$$F_{C_y}=-F_C\sin\theta=-F_C\frac{b}{a}=-3{,}546.814.973.09\cdot10^{22}\,\frac{149.483{,}055.858\cdot10^{6}}{149.504.10^{6}}=$$

$$=-3{,}546.318.099.14\cdot10^{22}\ [N] \qquad (32)$$

Note that forces $\bar{F}_C$ and $m\dot{\bar{v}}_C$ are collinear ; indeed :

$$\mathrm{tg}\,\beta_C=\frac{F_{C_x}}{F_{C_y}}=\frac{(m\dot{\bar{v}}_C)_x}{(m\dot{\bar{v}}_C)_y}=0{,}016.740.345.15 \qquad (33)$$

The module of force $m\dot{\bar{v}}_C$ is :

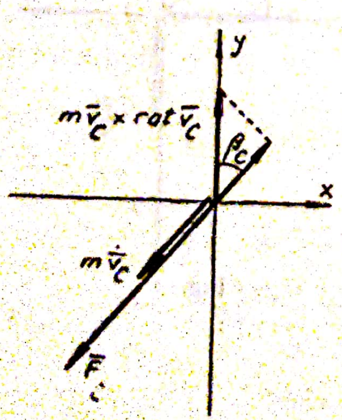

Fig. 5

$$|m\dot{\bar{v}}_C|=\frac{(m\dot{\bar{v}}_C)_x}{\sin\beta_C}=\frac{(m\dot{\bar{v}}_C)_x}{\mathrm{tg}\,\beta_C}\cdot\sqrt{1+\mathrm{tg}^2\beta_C}=$$

$$=10^{16}\cdot\frac{58.946{,}238.127.1}{0{,}016.740.345.15}\cdot\sqrt{1+1{,}674.034.515^2\cdot10^{-4}}=$$

$$=3{,}521.701.407.09\cdot10^{22}\ [N] \qquad (34)$$

On the account of the vectorial diagram of forces in point C, fig. 5, and of the eq. (1) we may write:

$$C_C=\frac{|\bar{F}_C-m\dot{\bar{v}}_C|}{|m\bar{v}_C\times\mathrm{rot}\,\bar{v}_C|\cos\beta_C}=\frac{|\bar{F}_C-m\dot{\bar{v}}_C|}{|m\bar{v}_C\times\mathrm{rot}\,\bar{v}_C|}\cdot$$

$$\cdot \sqrt{1+\mathrm{tg}^2\beta_{\cdot;}} = \frac{|-3,546.814.973.09+3,521.701.407.09|}{|3,522.194.830.49|} \cdot \tag{35}$$

$$\cdot 1,000.140.109.76 = 0,007.131.088.98.$$

For determination of constant C value at point E, we shall note with r and $\theta=30°$ the polar coordinates of point E. From eq. (4) we have. :

$$r = \frac{b^2}{a-c\cos 30} = \frac{2b^2}{2a-c\sqrt{3}} = \frac{2.149.483,055.958^2\cdot 10^{12}}{2\cdot 149.504\cdot 10^6-2502,397.952\cdot 10^6\sqrt{3}} =$$

$$= 151.660.514,878\cdot 10^3 \ [m] \tag{36}$$

The coordinates of point E, (x_E, y_E), are :

$$x_E = \frac{b^2\sqrt{3}}{2a-c\sqrt{3}}$$

$$y_E = \frac{b^2}{2a-c\sqrt{3}} \tag{37}$$

On the account of the equation (36), (37), (12), (15), (16), and (2) we may write :

$$\dot{m v}_E = -\bar{i}m\frac{\sqrt{3}}{8}\left(\frac{2\pi}{T}\right)^2\cdot\left(\frac{a}{b}\right)^4\frac{(2a-c\sqrt{3})^2}{a} - \bar{j}\frac{m}{8}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(2a-c\sqrt{3})^2}{a} =$$

$$= -\bar{i}\,5,975\cdot 10^{24}\frac{\sqrt{3}}{8}\cdot 3,942.410.189.64\cdot 10^{-14}\,.$$

$$\cdot\frac{(2\cdot 149.504\cdot 10^6-2502,397.952\cdot 10^6\cdot\sqrt{3})^2}{149.504\cdot 10^6}\cdot 1,000.560.556.99- \tag{38}$$

$$-\bar{j}\,\frac{5,975\cdot 10^{24}}{8}\,3,942.410.189.64\cdot 10^{-14}\cdot 1,000.560.556.99\,.$$

$$\cdot\frac{(2\cdot 149.504\cdot 10^6-2502,397.952\cdot 10^6\cdot\sqrt{3})^2}{149.504\cdot 10^6} =$$

$$= -\bar{i}\,2.963.764,795.76\cdot 10^{16} - \bar{j}\,1.711.130,402.66\cdot 10^{16}$$

$$\frac{(\dot{m v}_E)_x}{(\dot{m v}_E)_y} = \frac{2,963.764.795.76\cdot 10^{22}}{1,711.130.402.66\cdot 10^{22}} = 1,732\cdot 050.807.55 = \sqrt{3} \tag{39}$$

$$|\dot{m v}_E| = (\dot{m v})_x\cdot\frac{2}{\sqrt{3}} = 3,422.260.805.33\cdot 10^{22}\ [N] \tag{40}$$

$$\bar{m v}_E\times \mathrm{rot}\,\bar{v}_E = \bar{i}\frac{m}{4a}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4(a\sqrt{3}-2c)\,(2a-c\sqrt{3})+$$

$$+\bar{j}\frac{m}{4}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4(2a-c\sqrt{3})=$$

$$=\bar{i}\frac{5{,}975\cdot 10^{24}}{4\cdot 149.504\cdot 10^{6}}\cdot 3{,}942.410.189.64\cdot 10^{-14}\cdot 1{,}000.560.556.99\cdot$$

$$\cdot(149.504\cdot 10^{6}\cdot\sqrt{3}-2\cdot 2.502{,}397.952\cdot 10^{6})\cdot(2\cdot 149.504\cdot 10^{6}- \tag{41}$$

$$-2.502{,}397.952\cdot 10^{6}\sqrt{3})+\bar{j}\frac{5{,}975\cdot 10^{24}}{4}3{,}942.410.189.64\cdot 10^{-14}\cdot$$

$$\cdot 1{,}000.560.556.99\cdot(2\cdot 149.504-2.502{,}397.952\cdot\sqrt{3})\cdot 10^{6}=$$

$$=\bar{i}\,2.949.233{,}709.52\cdot 10^{16}+\bar{j}\,1.736.298{,}982.22\cdot 10^{16}$$

$$\frac{(m\bar{v}_E\times \operatorname{rot}\bar{v}_E)_x}{(m\bar{v}_E\times \operatorname{rot}\bar{v}_E)_y}=\frac{2.949.233{,}709.52}{1.736.298{,}982.22}=1{,}698.574.807.51=\operatorname{tg}\beta_E \tag{42}$$

$$|m\bar{v}_E\times \operatorname{rot}\bar{v}_E|=\frac{(m\bar{v}_E\times \operatorname{rot}\bar{v}_E)_x}{\operatorname{tg}\beta_E}\cdot\sqrt{1+\operatorname{tg}^2\beta_E}= \tag{43}$$

$$=2.949.233{,}709.52\cdot 10^{16}\frac{\sqrt{1+1{,}698.574.807.51^2}}{1{,}698.574.807.51}=3{,}422.384.202.41.\cdot 10^{22}\ [N]$$

$$\beta_E=59°30'48'' \tag{44}$$

$$F_E=K\frac{M\,m}{r^2}=6{,}664\cdot 10^{-11}\frac{1{,}991\cdot 10^{30}\cdot 5{,}975\cdot 10^{24}}{151.660{,}514.878^2\cdot 10^{12}}= \tag{45}$$

$$=3{,}446.\cdot 665.253.19\cdot 10^{22}\ [N]$$

$$(\bar{F}_E)_x=|\bar{F}_E|\cos 30=F_E\cdot\frac{\sqrt{3}}{2} \tag{46}$$

$$(\bar{F}_E)_y\ |\bar{F}_E|\sin 30=F_E\cdot\frac{1}{2} \tag{47}$$

$$\frac{(\bar{F}_E)_x}{(\bar{F}_E)_y}=\frac{(m\dot{\bar{v}}_E)_x}{(m\dot{\bar{v}}_E)_y}=\operatorname{tg}60° \tag{48}$$

The angle between the force $\bar{F}_E$ and $m\bar{v}_E\times \operatorname{rot}\bar{v}_E$ is

$$(60-\beta_E)=29'12'' \tag{49}$$

Taking into account the vectorial diagram of the forces at point E, fig. 6, the equation (1) may be written as under:

$$C_E=\frac{|\bar{F}_E-m\dot{\bar{v}}_E|}{|m\bar{v}_E\times \operatorname{rot}\bar{v}_E|\cos(60-\beta_E)}=$$

$$= \frac{|\,3{,}446.665.253.19\cdot 10^{22} - 3{,}422.260.805.33\cdot 10^{22}\,|}{|\,3{,}422.384.202.41\cdot 10^{22}\,|\ 0{,}999.964} = \qquad (50)$$

$$= 0{,}007.131.089.02$$

At the point D, since

$$y=0,\quad \theta=\pi,\quad x=-r=-(a-c), \qquad (51)$$

from equations (12), (15), (16) and (2) we immediately obtain :

$$m\dot{\bar v}_D = \bar i\, m\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a+c)^2}{a} = -m\bar v_D\times \mathrm{rot}\,\bar v_D =$$

$$= \bar i\, 5{,}975\cdot 10^{24} 3{,}942.410.189.64\cdot 10^{-14}\cdot$$

$$\cdot 1{,}000.560.556.99\cdot \frac{152.006{,}397.952^2\cdot 10^{12}}{139.504\cdot 10^{6}} =$$

$$= \bar i\, 3{,}642.621.276.83\cdot 10^{22}\ [N] \qquad (54)$$

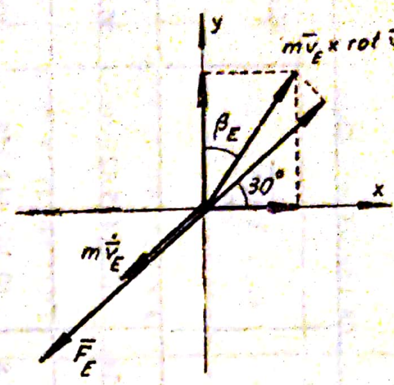

Fig. 6

$$F_D = K\frac{M\,m}{(a-c)^2} = 6{,}664\cdot 10^{-11}\cdot \frac{1{,}991\cdot 10^{30}\cdot 5{,}975\cdot 10^{24}}{147.001{,}602.048^2\cdot 10^{12}} = \qquad (51)$$

$$= 3{,}668.597.134.22\cdot 10^{22}\ [N]$$

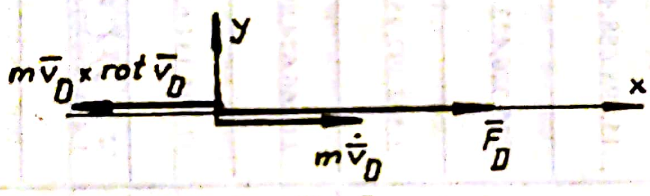

Fig. 7

According to the vectorial diagram of the forces in D, fig. 7, we have :

$$C_D = \frac{|\,\bar F_D - m\dot{\bar v}_D\,|}{|\,m\bar v_D\times \mathrm{rot}\,\bar v_D|} = \frac{|\,3{,}668.597.134.22\cdot 10^{22} - 3{,}642.621.276.83\cdot 10^{22}\,|}{|\,-3{,}642.621.276.83\cdot 10^{22}\,|} =$$

$$= 0{,}007.131.089.23 \qquad (52)$$

If the above stated equations are applied to the other planets of the Sun system, with the parameters of Table 1, we obtain the same numerical value for the universal constant C, this being clearly a full confirmation of the general law of universal gravity which we have established.

We shall not discuse here in the deep implications of this new fundamental law, which obliges us to reexamine our thought system for correct understanding of matter properties, but we dare to underline the decisive importance of this law, which, in our opinion, opens inimaginable prospects for the scientific knowledge. In an another paper we shall extent this law to the hydrogen atom quantified by N. Bohr.

Table 1

$M_s = 1{,}991\ 10^{30}$ [kg] ; $2\pi = 62.831.853{,}07 \cdot 10^{-7}$

Planet	Major semiaxis a [km]	Eccentricity $\frac{c}{a}$	Focal semidistance c [km]	$\sqrt{a+c}$ [$km^{1/2}$]	$\sqrt{a-c}$ [$km^{1/2}$]	Minor semiaxes b [km]
Mercury	57.872.998,4	0,2056	11.898.688,471	8.352,944.802.34	6780,435.821.46	56.636.606,1524
Venus	108.136.243,2	0,0068	735.326,453.76	10.434,153.997.8	10.363,441.356	108.133.743,055
Earth	149.504.000	0,0167	2.502.397,952	12.329,087.474.3	12.124,421.719.8	149.483.055,958
Mars	227.694.592	0,0934	21.266.674,8928	15.778,506.484.7	14.367,599.599.4	226.699.262,817
Jupiter	777.570.304	0,0481	37.401.131,6224	28.547,704.559.5	27.206,050.289.7	776.670.285,901
Saturn	1.426.000.000	0,0530	75.578.000 ~~1.423.995.774,53~~	38.750,199.999.4	36.748,088.385.6	1.423.995.774.53
Uranus	2.871.971.840	0,0482	138.429.042,688	54.867,120.233.1	52.283,293.673.2	2.868.633.760,14
Neptune	4.492.595.200	0,0054	24.260.014,08	67.207,553.251.6	66.845,607.080.1	4.492.529.697,47
Pluto	5.900.000.000	0,251	1.480.900.000	85.912,164.447.1	66.476,311.570.3	5.711.123.811,46

Tabel 1 (continuation)

Planet	$\frac{a}{b}$	$\left(\frac{a}{b}\right)^2$	$\left(\frac{a}{b}\right)^4$	period T $[s]$	$\left(\frac{2\pi}{T}\right)^2$ $[s^{-2}]$	Mass [kg]
Mercury	1,021.830.267.23	1,044.137.095.02	1,090.222.273.19	7.621.381,842.61	$67,966.091.329.3.10^{-14}$	$3,2863.10^{23}$
Venus	1,000.023.120.85	1,000.046.242.23	1,000.092.486.59	19.465.997,4273	$10,418.529.203.5.10^{-14}$	$4,8696.10^{24}$
Earth	1,000.140.109.8	1,000.280.239.23	1,000.560.556.99	31.644.552,9028*)	$3,942.410.189.64.10^{-14}$	$5,975.10^{24}$
Mars	1,004.390.526.77	1,008.800.330.26	1,017.678.106.33	59.476.967,716	$1,115.994.614.18.10^{-14}$	$6,42495.10^{23}$
Jupiter	1,001.158.816.18	1,002.318.975.21	1,004.643.328.06	375.343.798,406	$2,802.213.669.96.10^{-16}$	$1,901.10^{27}$
Saturn	1,001.407.465.88	1,002.816.912.72	1,005.641.760.43	932.178.396,108	$45,431.980.512.4.10^{-18}$	$5,690.278.10^{26}$
Uranus	1,001.163.647.97	1,002.328.650.01	1,004.662.722.63	2.664.342.247,52	$5,561.343.416.23.10^{-18}$	$8,682.751.10^{25}$
Neptune	1,000.014.580.32	1,000.029.160.85	1,000.058.322.55	5.212.735.375,22	$1,452.875.430.32.10^{-18}$	$10,337272.10^{25}$
Pluto	1,033.071.632.62	1,067.236.998.12	1,138.994.810.15	7.845.086.975,77	$64,145.206.995.10^{-20}$	$1,0988329.10^{24}$

*) 1 sideral year = 365,256.360.4 × $24^h\ 3m\ 56^s,5554$ = 31.644.552,9028 [s].

ASUPRA LEGII GRAVITAŢIEI UNIVERSALE

(Rezumat)

Abandonînd concepţia heliocentrică, această lucrare pune bazele concepţiei vegacentrice plecînd de la ideea fundamentală că asupra oricărei particule sau corp liber, care se deplasează într-un cîmp gravific, acţionează o forţă care are modulul dependent de o forţă de deviere laterală, care este proprie structurii Universului. Completînd legea gravitaţiei universale a lui Newton cu această forţă, se obţine o ecuaţie care descrie mişcarea soarelui şi a planetelor sale spre steaua Vega, de unde rezultă o constantă universală, adimensională, care păstrează aceeaşi valoare numerică în orice punct al traiectoriilor descrise de planete, ceea ce reprezintă o confirmare a legii stabilită de noi.

SUR LA LOI DE LA GRAVITATION UNIVERSELLE

(Résumé)

Abandonnant la conception héliocentrique, cet ouvrage pose les bases de la conception végacentrique en partant de l'idée fondamentale que sur toute particule ou corps libre qui se déplace dans un champ gravifique actionne une force qui a le module dépendant d'une force de déviation latérale, qui appartient à la structure de l'Univers. Complétant la loi de la gravitation universelle de Newton avec cette force, on obtient une équation qui décrit le mouvement du Soleil et de ses planètes, vers l'étoile Véga, d'où résulte une constante universelle, adimensionnelle, qui garde, pour n'importe quelle planète, la même valeur dans tous les points des trajectoires des planètes, ce qui répresente une confirmation de la loi générale établie par nous.

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NEW LAWS OF THE SOLAR PLANETARY SYSTEM

BY

ALEXANDRU VASILESCU[1] and MIHAIL VASILESCU**

Let us consider the implicit function:

$$f(|\dot{\bar{v}}|,\ r,\ M,\ K,\ T\ |\mathrm{rot}\ \bar{v}|,\ \dot{\theta},\ v)=0 \tag{1}$$

describing the movement of the planets around the Sun, where:

$|\dot{\bar{v}}|$ is the module of planet acceleration
r — the module of vector radius of planet
M — the mass of the Sun
K — the constant of universal gravity
T — the revolution period of planet around Sun
$|\mathrm{rot}\ \bar{v}|$ — the planet velocity curl
$\dot{\theta}$ — the angle speed of planet
v — the module of planet velocity

Applying to function (1) the theorem π, we obtain the following basic system of non-dimensional complexes:

$$\pi_1=\frac{KM}{r^2|\dot{\bar{v}}|};\quad \pi_2=T\sqrt{\frac{|\dot{\bar{v}}|}{r}};\quad \pi_3=|\mathrm{rot}\ \bar{v}|\sqrt{\frac{r}{|\dot{\bar{v}}|}} \tag{2}$$

$$\pi_4=\dot{\theta}\sqrt{\frac{r}{|\dot{\bar{v}}|}};\quad \pi_5=\frac{v}{\sqrt{r|\dot{\bar{v}}|}}.$$

We shall check further whether everyone of these non-dimensional complexes maintain for all the planets the same numerical value in all the points or in one point only of the projection in the ecliptic plane of the planets trajectories; the non-dimensional complexes complying with these conditions represent the laws of our solar system. We shall notice that the first non-dimensional complex maintains the same numerical value for all

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the planets in every point of the projection in the ecliptic plane of the planets trajectories.

The first law of the solar system, obtained in this way, may be written therefore as :

$$\frac{KM}{r^2 |\dot{\bar{v}}|} = 1 + C \tag{3}$$

where C is the dimensionaless constant of the mathematical relationship of the side deviating force that characterizes the movement of planets in our solar system space.

Writing :

$$r^2 |\dot{\bar{v}}| = r^2 \cdot r\dot{\theta} |\text{rot } \bar{v}| = \frac{KM}{1+C} \tag{4}$$

and taking into account that

$$r^2 \dot{\theta} = \frac{2\pi}{T} ab \tag{5}$$

$$|\text{rot } \bar{v}| = \frac{2\pi}{T} \cdot \frac{a^2}{br} \tag{6}$$

it follows

$$\left(\frac{2\pi}{T}\right)^2 \cdot a^3 = \frac{KM}{1+C} \tag{7}$$

which represents the III-d law of Kepler.

The second complex, π_2, maintains, for every planet, the same numerical value in point C, fig. 2, where

$$r = r_C = a \text{ and } |\dot{\bar{v}}_C| = \left(\frac{2\pi}{T}\right)^2 \cdot a ;$$

indeed, if we write the complex π_2 in point C we obtain the following law :

$$T \sqrt{\frac{|\dot{\bar{v}}_C|}{r_C}} = 2\pi. \tag{8}$$

Taking into account the equation (3), the complex π_2 may be written as :

$$\pi_2 = T \sqrt{\frac{|\dot{\bar{v}}_C|}{r_C}} = T \sqrt{\frac{1}{r_C} \cdot \frac{KM}{(1+C) r_C^2}} = 2\pi, \tag{9}$$

which for $r = r_C = a$ leads to the III-d law of Kepler.

The complexes π_3 and π_4 are equivalent and maintain in point B of fig. 2 the same numerical value for every planet.

According to equations (5) and (6) we have :

$$|\text{rot } \bar{v}| = \frac{ar}{b^2} \dot{\theta} , \tag{10}$$

which for $r=r_B=\dfrac{b^2}{a}$ gives $|\operatorname{rot}\bar{v}_B|=\dot{\theta}_B$;

therefore we have the following law:

$$\pi_3=\pi_4=\dot{\theta}_B\sqrt{\frac{r_B}{|\dot{\bar{v}}_B|}}=\dot{\theta}_B\sqrt{\frac{r_B}{r_B\dot{\theta}_B|\operatorname{rot}\bar{v}_B|}}=1 \tag{11}$$

Taking into account the equations (3) and

$$|\dot{\bar{v}}_B|=\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4 a, \tag{12}$$

we may write successively:

$$\pi_3=\pi_4=|\operatorname{rot}\bar{v}_B|\left(\frac{KM}{1+C}\right)^{\frac{1}{4}}|\dot{\bar{v}}_B|^{-\frac{3}{4}}=$$

$$=\frac{2\pi}{T}\frac{a^2}{b\frac{b^2}{a}}\left(\frac{KM}{1+C}\right)^{\frac{1}{4}}\left[\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4 a\right]^{-\frac{3}{4}}=\left(\frac{2\pi}{T}\right)^{-\frac{1}{2}}a^{-\frac{3}{4}}\left(\frac{KM}{1+C}\right)^{\frac{1}{4}}=1 \tag{13}$$

The last equality represents the III-d law of Kepler. The last complex, π_5, maintains, for every planet, the same numerical value in point C, fig. 2, where:

$$r=r_C=a,\ v_C=\frac{2\pi}{T}\ a,\ |\dot{\bar{v}}_C|=\left(\frac{2\pi}{T}\right)^2\cdot a;$$

indeed, in the point C we obtain the following law:

$$\frac{v_C}{\sqrt{r_C|\dot{\bar{v}}_C|}}=1 \tag{14}$$

Taking into account the equation (3), the equation (14) takes the form:

$$\frac{v_C}{\sqrt{r_C\dfrac{KM}{(1+C)r_C^2}}}=1 \tag{15}$$

which for $r=r_C=a$ and $v_C=\dfrac{2\pi}{T}\cdot a$ leads to the III-d law of Kepler.

Since to every physical phenomenon corresponds an infinite number of basic systems of non-dimensional complexes, we conclude that the laws of solar system, maintaining the same structure, may be written in an infinite number of mathematical forms.

Using the four laws represented on the equasions (3), (8), (11), (14), we may write following relationships:

$$\left(\frac{T_n}{T_m}\right)^2 = \left(\frac{a_n}{a_m}\right)^3 = \left(\frac{v_m}{v_n}\right)_C^6 = \left(\frac{\dot{\bar{v}}_m}{\dot{\bar{v}}_n}\right)^{\frac{3}{2}} = \left(\frac{\dot{\theta}_m}{\dot{\theta}_n}\right)_B^2 \cdot \left(\frac{a_n}{a_m}\right)^6 \cdot \left(\frac{b_m}{b_n}\right)^6 \tag{16}$$

which complete the kinematics of our solar system.

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NOI LEGI ALE SISTEMULUI PLANETAR SOLAR

(Rezumat)

Aplicind teoria similitudinii in cazul mișcării tridimensionale a planetelor, se deduc patru legi ale sistemului planetar solar.

DES NOUVELLES LOIS DU SYSTÈME PLANÉTAIRE SOLAIRE

(Résumé)

Appliquant la théorie de la similitude au cas du mouvement tridimensionnel, des planètes, on obtient quatre lois du système planétaire solaire.

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ABOUT THE RATIO BETWEEN INERT AND GRAVIFIC MASS

BY

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Let be written the universal attraction law as under :

$$m_i\,|\dot{\bar{v}}| = K\frac{Mm_g}{r^2} \tag{1}$$

$$\frac{KM}{r^2\,|\dot{\bar{v}}|} = \frac{m_i}{m_g} \tag{2}$$

As in every point of the trajectory of every planet we have :

$$\frac{KM}{r^2\,|\dot{\bar{v}}|} = 1+C = 1{,}007.131.089.24 \tag{3}$$

it follows that the law of gravity given by Newton leads us to the relationship :

$$\frac{m_i}{m_g} = 1+C = 1{,}007.131.089.24 \tag{4}$$

Using the law :

$$m_i\,|\dot{\bar{v}}| = K\frac{Mm_g}{r^3}\bar{r} + [(Cm_i\,\bar{v}\times\mathrm{rot}\,\bar{v})\,\bar{u}_0]\,\bar{u}_0 \tag{5}$$

written in two certain points, for examples A and B, fig. 2 :

$$[(C\bar{v}_A\times\mathrm{rot}\,\bar{v}_A)\,\bar{u}_0]\,\bar{u}_0 = m_i\dot{\bar{v}}_A - K\frac{Mm_g}{r_A^3}\bar{r}_A \tag{6}$$

$$[(C\bar{v}_B\times\mathrm{rot}\,\bar{v}_B)\,\bar{u}_0]\,\bar{u}_0 = m_i\dot{\bar{v}}_B - K\frac{Mm_g}{r_B^3}\bar{r}_B \tag{7}$$

and admitting that ratio m_i/m_g has the same value in A and B, by division we obtain :

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$$\frac{|[(\bar{v}_A \times \operatorname{rot} \bar{v}_A)\, \bar{u}_0]\, \bar{u}_0|}{|[(\bar{v}_B \times \operatorname{rot} \bar{v}_B)\, \bar{u}_0]\, \bar{u}_0|} = \frac{\left| m_i \dot{\bar{v}}_A - K \frac{M m_g}{r_A^3} \bar{r}_A \right|}{\left| m_i \dot{\bar{v}}_B - K \frac{M m_g}{r_B^3} \bar{r}_B \right|} \tag{8}$$

whence finally results:

$$\frac{m_i}{m_g} = 1 + C \tag{9}$$

By calculating the ratio $\frac{m_i}{m_g}$ in any other pair of points, we obtain — after eliminating beforehand of C, as we made above — the same value for this ratio as in the case when for this we use the equation (2).

We notice that for $C = 0{,}007.131.089.24$, the equation (5) leads to ratio:

$$\frac{m_i}{m_g} = 1 \tag{10}$$

In other words, from the universal gravity law of N e w t o n, results:

$$\frac{m_i}{m_g} = 1 + C$$

and if this law is completed with a side deviating force it follows:

$$\frac{m_i}{m_g} = 1$$

As may be seen, the ratio $\frac{m_i}{m_g}$ is dependent on the mass of the Sun.

ASUPRA RAPORTULUI DINTRE MASA INERTĂ ȘI MASA GRAVIFICĂ

(Rezumat)

Se arată că în cazul mișcării plane a planetelor avem $\frac{m_i}{m_g} \neq 1$, iar în cazul mișcării tridimensionale avem $\frac{m_i}{m_g} = 1$.

SUR LE RAPPORT ENTRE LA MASSE INERTE ET LA MASSE GRAVITATIONNELLE

(Résumé)

On montre que pour le mouvement plan des planètes résulte $\frac{m_i}{m_g} \neq 1$ et pour le mouvement tridimensionnel $\frac{m_i}{m_g} = 1$.

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THE PRECESSION OF MERCURY PLANET PERIHELION CONFRONTED WITH A PROBLEM OF SIMILITUDE

BY

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Let be considered the universal gravity written as follows:

$$\frac{KM}{r^2|\dot{\bar{v}}|}=\frac{m_i}{m_g}=1+C, \tag{1}$$

whence th law follows:

$$r^2\,|\dot{\bar{v}}|=\frac{KM}{1+C}, \tag{2}$$

which is valid in every point of the trajectory of any planet.

If $m_i=m_g$ is assumed, the eq. (1) leads to the law:

$$r^2\,|\dot{\bar{v}}|=KM, \tag{3}$$

which is valid only in the points of ellipse, characterized by $r=a$, as are the points C and its symmetrical against the x — axis, fig. 2

Since in point C, fig. 2, we hawe:

$$r=a$$

$$|\dot{\bar{v}}|=\left(\frac{2\pi}{T}\right)^2\cdot a \tag{4}$$

the equations (2) and (3) take the form:

$$a^3\left(\frac{2\pi}{T}\right)^2=\frac{KM}{1+C} \tag{5}$$

$$a^3\left(\frac{2\pi}{T}\right)^2=KM \tag{6}$$

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The equation (6) is the III-d law of K e p l e r, written for the case of planet, describing around the Sun, a plane trajectory; we shall call this kind of movement as „movement on model" of the M e r c u r y planet and the three — dimensional movement of the planet, to which the eq. (5) corresponds, we shall call „movement in nature" of M e r c u r y planet.

As the movement of the planet is a periodical phenomenon, the confrontation between the model and nature should be made, according to Similitude Theory, in the corresponding time moments only.

For analyzing this problem, we shall consider the implicit function:

$$f(|\dot{\bar{v}}|,\ r,\ M,\ K,\ T,\ |\operatorname{rot}\bar{v}|,\ \dot{\theta},\ v)=0 \tag{7}$$

(which is identical to eq. (1) from the paper „New laws of the solar planetary system") and shall write the model law corresponding to this function.

It must be emphasized that, for this purpose, we have to keep from the similitude criterions, corresponding to function (7), only those criterions which maintain in a point of the planet trajectory, numerical values which are the same both for the model and the nature.

We have seen that this condition is satisfied in point C, fig. 2, by the following criterions:

$$r^2\,|\dot{\bar{v}}_C|=\frac{KM}{1+C};\quad T\sqrt{\frac{|\dot{\bar{v}}_C|}{r_C}}=2\pi$$
$$\frac{v_C}{\sqrt{r_C\,|\dot{\bar{v}}_C|}}=1 \tag{8}$$

To be able to write the model law corresponding to critierions (8), we must know first of all if „the movement on model" of the M e r c u r y planet has been or not distorted (deformed) or — in other words — we must know whether the model has been „carried out" at one or two scales.

If the periods on model and nature are noted by:

$$T_m=\frac{2\pi a_m^{\frac{3}{2}}}{\sqrt{KM}};\quad T_n=\frac{2\pi a_n^{\frac{3}{2}}\sqrt{1+C}}{\sqrt{KM}}, \tag{9}$$

the time scale will be:

$$K_T=\frac{T_n}{T_m}=\left(\frac{a_n}{a_m}\right)^{\frac{3}{2}}\sqrt{1+C} \tag{10}$$

Since to each period corresponds another ellipse, for $T_n=$ $=7.621.381{,}842.61$ [s] was calculated (Table 1):

$$a_n=57.872.998{,}4\ \text{[Km]}$$
$$b_n=56.636.606{,}1524\ \text{[Km]} \tag{11}$$
$$c_n=11.898.688{,}471\ \text{[Km]}$$

In order to find the homolog magnitudes on model, we shall outgo from the value, determined „in nature", by experimental way, for the perihelion precession of Mercury planet.

Noting with $\bar{r}(t)$ the vector radius of the planet and aasuming as intermediary argument, the length of curve arc, $s(t)$, measured from an ~~from an~~ origin arbitrarily selected on the curve, we may write :

$$\frac{d\bar{r}}{dt}=\frac{d\bar{r}}{ds}\cdot\frac{ds}{dt}=\bar{\tau}v=\bar{v} \qquad (12)$$

where :

$\bar{v}$ is the instantaneous speed of planet, directed on the tangent to trajectory

$\bar{\tau}=\frac{d\bar{r}}{ds}$ is the unit vector of speed vector $\bar{v}$,

$v=\frac{ds}{dt}=\lim\limits_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$ is the module of the instantaneous speed.

As the experimental value of perihelion precession of Mercury planet is 42,9 seconds of the arc per century, the length of curve arc described by the planet at perihelion during the Δt time is (Table 1) :

$$\Delta s=(a-c)\frac{2\pi\cdot 0,429}{360\cdot 3\,600}=$$

$$=45.974.309,929\frac{62.831.853,07\cdot 10^{-7}\cdot 0,429}{360\cdot 3\,600}=95,619.700.314.9\ [\text{Km}] \qquad (13)$$

The speed of the planet at perihelion is (fig. 2) :

$$v_D=\frac{2\pi}{T}\cdot\frac{a}{b}\cdot(a+c) \qquad (14)$$

$= T_n$)

where T is the period of Mercury, given in Table 1 and obtained from eq. (5) for

$$a_n=0,3871\cdot 149.504.000=57.872.998,4\ [\text{Km}]$$

With the data from the Table 1, we may write :

$$v_D=8,244.154.979.69\cdot 10^{-7}\cdot 1,021.830.267.23\cdot 69.771.686,871=$$

$$=58,776.555.720.4\left[\frac{\text{Km}}{\text{s}}\right] \qquad (15)$$

$$\Delta t\simeq\frac{\Delta s}{v_D}=\frac{95,619.700.314.9}{58,776.555.720.4}=1,626.834.018.13\left[\frac{\text{s}}{\text{revol}}\right] \qquad (16)$$

With these data we may write successively

$$T_m=T_n-1,626.834.018.13=7.621.380,215.78\ [\text{s}] \qquad (17)$$

$$K_T = \frac{7.621.381,842.61}{7.621.380,215.78} = 1,000.000.213.45 = K_a^{\frac{3}{2}} \sqrt{1+C} \tag{18}$$

$$K_a^{\frac{3}{2}} = \frac{1,000.000.213.45}{\sqrt{1,007.131.089.23}} = 0,996.453.625.14 \tag{19}$$

$$K_a = 0,997.634.350.47 = \frac{a_n}{a_m} \tag{20}$$

$$a_m = \frac{a_n}{K_a} = \frac{57.872.998,4}{0,997.634.350.47} = 58.010.230,2739 \quad [\mathrm{Km}] \tag{21}$$

Assuming that on the model the eccentricity of the ellipse has the same value : $\frac{c}{a} = 0,2056$, as in the nature, Table 1, we have :

$$\begin{gathered}
c_m = a_m \cdot 0,2056 = 11.926.903,3443 \ [\mathrm{Km}] \\
a_r + c_m = 69.937.133,6182 \ [\mathrm{Km}] \\
\sqrt{a_m + c_m} = 8.362,842.436.52 \ [\mathrm{Km}^{\frac{1}{2}}] \\
a_m - c_m = 46.083.326,929.6 \ [\mathrm{Km}] \\
\sqrt{a_m - c_m} = 6.788,470.146.47 \ [\mathrm{Km}^{\frac{1}{2}}] \\
b_m = \sqrt{(a_m + c_m)(a_m - c_m)} = 56.770.906,219.9 \ [\mathrm{Km}] \\
K_b = \frac{b_n}{b_m} = \frac{56.636.606,1524}{56.770.906,219.9} = 0,997.634.350.47
\end{gathered} \tag{22}$$

Having :

$$K_a = K_b = K_c \tag{23}$$

it follows that the „movement on model" of Mercury planet is a model made at a single scale and thus the law of the model corresponding to criterions (8) for $r_c = a$ is :

$$\begin{gathered}
K_a^2 K_{|\dot{\vec{v}}_c|} = \frac{1}{1+C}; \quad K_T \sqrt{\frac{K_{|\dot{\vec{v}}_c|}}{K_a}} = 1 \\
\frac{K_{v_c}}{\sqrt{K_a K_{|\dot{\vec{v}}_c|}}} = 1,
\end{gathered} \tag{24}$$

whence we have :

$$K_{|\dot{v}_c|} = \frac{|\dot{v}_{c_n}|}{|\dot{v}_{c_m}|} = \frac{1}{K_a^2(1+C)}$$

$$K_{v_C} = \frac{v_{C_n}}{v_{C_m}} = \frac{1}{\sqrt{K_a(1+C)}} \qquad (25)$$

$$K_T = \frac{T_n}{T_n} = K_a^{\frac{3}{2}}\sqrt{1+C}$$

It may be seen that the scale of periods, obtained in this way, is identical to relationship (18).

In point C as we have :

$$v_C = \frac{2\pi}{T}\cdot a \,; \quad |\dot{v}_C| = \left(\frac{2\pi}{T}\right)^2 \cdot a, \qquad (26)$$

the numerical check up of the first two relationship (25) is imediately available.

Therefore we have to point out that, if on the model orbit, the Mercury planet comes again to perihelion after T_m seconds, on the real orbit the planet will be back at perihelion, according to similitude theory, after a greater time interval, which is given by the corresponding period in nature :

$$T_n = T_m + 1{,}626.834.018.13 \ [s] \,; \qquad (27)$$

this means, in other worlds, that the precession phenomenon of planets perihelion is not taking place, the corresponding times on the real orbit and model orbit explaining the a supposed advance of perihelion.

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PRECESIA PERIHELIULUI PLANETEI MERCUR CONFRUNTATĂ CU O PROBLEMĂ DE SIMILITUDINE

(Rezumat)

Plecînd de la perioada dată de legea III-a a lui Kepler pentru mişcarea plană şi pentru mişcarea tridimensională a planetelor, în cadrul teoriei similitudinii se stabileşte legătura între timpul pe model şi timpul pe natură, subliniindu-se că fenomenul de precesie a periheliului planetelor nu are loc.

LA PRÉCESSION DU PÉRIHÉLIE DE LA PLANÈTE MERCURE CONFRONTÉE AVEC UN PROBLÈME DE SEMILITUDE

(Résumé)

En partant de la période donnée par la troisième loi de Kepler pour le mouvement plan et pour le mouvement tridimensionnel des planètes, utilisant la théorie de la similitude on établie la liaison entre „le temps sur le modèle" et „le temps sur la nature", en soulignant que le phénomène de la précession du périhélie n'existe pas.

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STUDY OF MOON MOVEMENT BY MEANS OF UNIVERSAL GRAVITY LAW COMPLETED WITH SIDE DEVIATING FORCE

BY

AL. A. VASILESCU* and M. AL. VASILESCU

We consider the Universal Gravity Law, completed with side deviating force, written as follows:

$$m\dot{\bar{v}} = -K\frac{Mm}{r^2}\bar{r}_0 + [(Cm\bar{v}\times \mathrm{rot}\,\bar{v})\bar{r}_0]\bar{r}_0. \tag{1}$$

We intend to use (1)to investigate the Moon movement within the solar system space.

However, to be able to make investigations on Sun-Earth-Moon system movement, we shall analyse first, outgoing from eq. (1), the movement of the Moon around the Earth, considering the latter as having one translation movement only along axis z, which is the apex direction.

In case of this movement, the trajectory described by the Moon around the Earth is a three-dimensional curve, having as projection on ecliptic plane an ellipse, with Earth located in one of its focal points.

As the angle between Moon orbit and ecliptic plane is very small, ab. 5°9′, we shall investigate the movement of the Moon against a right-handed system ($\bar{i}\times\bar{j}=\bar{k}$), linked to Vega star, with the plane Oxy located in the ecliptic plane, considered at a given time and with the origin O in the focus of the ellipse, where Earth is located.

The Ox — axis is directed along the big axis of the ellipse, to the apogee and Oy — axis is perpendicular to x — axis. The Oz — axis is normal to xy and represents the Earth moving direction in its translation movement to Vega star.

We shall further make use of the same notations and formulae as we did in the paper „*About The Law Of Universal Gravity*".

In eq. (1) the notations represent therefore:

$m=7{,}349\cdot 10^{22}$ kg — mass of the Moon,

$\bar{v}$ — speed of the Moon, tangential to its helicoidal trajectory,

rot $\bar{v}$ — the Moon velocity curl,

$\bar{r}_0$ — the unit vector of the force $\bar{F}$,

$$\bar{F}=k\frac{Mm}{r^2}\bar{r}_0.$$

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The trajectory of the Moon projected on ecliptic plane is an ellipse having the equation:

$$r(\theta)=\frac{p}{1-e\cos\theta} \tag{2}$$

where:

r, θ are the vector radius and the polar angle of the ellipse, respectively.

e — ellipse eccentricity,

a, b, c — major semiaxis, minor semiaxis and focal semidistance of ellipse.

$p=\frac{b^2}{a}=a(1-e^2)$ — a parameter of the ellipse.

The speed of the Moon may be decomposed in two components:

$$\bar{v}=\bar{w}+\bar{v}_0. \tag{3}$$

The component $\bar{w}$, having constant size and direction, is orientated along z — axis and represents the translation speed of the whole solar system towards Vega star, Fig. 1.

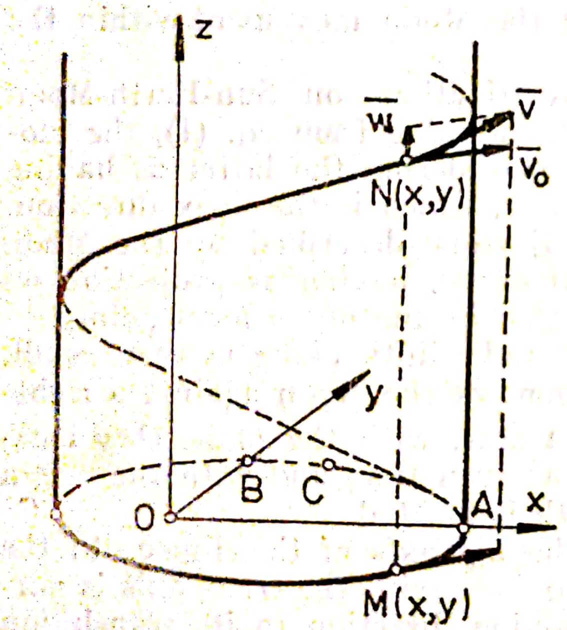

Fig. 1

The component $\bar{v}_0$, of variable size, is located in the plane of the ecliptic and is tangential to the ellipse described by eq. (2).

By deriving the coordinates of the Moon projection on ecliptic plane:

$$x=r(\theta)\cos\theta$$
$$y=r(\theta)\sin\theta \tag{4}$$

we obtain the following equations:

$$\dot{\theta}=\frac{2\pi}{T}\left(\frac{a}{b}\right)^3(1-e\cos\theta)^2$$
$$\dot{x}=-\frac{2\pi}{T}\frac{a^2}{b}\frac{y}{r} \tag{5}$$

$$\dot{y}=\left(-\frac{b^2c}{a^2}+\frac{b^2x}{ar}\right)\frac{2\pi}{T}\left(\frac{a}{b}\right)^3 \tag{6}$$

$$\ddot{x}=-\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c\cos\theta)^2}{a}\cos\theta$$
$$\ddot{y}=-\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c\cos\theta^2)}{a}\sin\theta. \tag{7}$$

Using these equations we obtain:

$$\bar{v}_0=\bar{i}\dot{x}+\bar{j}\dot{y}=-\bar{i}\frac{2\pi}{T}\cdot\frac{a^2}{b}\cdot\frac{y}{r}+\bar{j}\left(-\frac{b^2c}{a^2}+\frac{b^2x}{ar}\right)\frac{2\pi}{T}\left(\frac{a}{b}\right)^3 \tag{8}$$

$$\dot{\bar{v}}_0 = \bar{i}\ddot{x} + \bar{j}\ddot{y} = -\bar{i}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4 \frac{(a-c\cos\theta)^2}{a}\cos\theta - -\bar{j}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)\frac{(a-c\cos\theta)^2}{a}\sin\theta \tag{9}$$

$$\text{rot}\ \bar{v}_0 = \bar{k}\frac{2\pi}{T}\frac{a^2}{br}. \tag{10}$$

Since $\dot{\bar{v}} = \dot{\bar{v}}_0 \quad (\bar{w} = ct)$. (11)

and taking into account that in the ecliptic plane the vectors $\bar{v}$ and $\dot{\bar{v}}$ have, respectively, the same projections as the vectors $\bar{v}_0$ and $\dot{\bar{v}}_0$, given by the eq. (6) and (7), we may write:

$$\bar{v}\times\text{rot}\ \bar{v} = \bar{i}\left(\frac{2\pi}{T}\right)^2\left[-\left(\frac{a}{b}\right)^3\frac{bc}{r} + \left(\frac{a}{b}\right)^2\frac{a^2}{r^2}x\right] + \bar{j}\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^2\frac{a^2}{r^2}y \tag{12}$$

From III[a] law of Kepler, written for the three-dimensional movement of planets:

$$K\frac{M}{1+C} = \left(\frac{2\pi}{T}\right)^2 a^3. \tag{13}$$

where

$$K = 6{,}664\cdot 10^{-11}\left[\frac{m^3}{\text{kg}\ s^2}\right]$$

$$M = 5{,}975\cdot 10^{24}\ [\text{kg}]\ \text{(mass of the Earth)}$$

$$C = 0{,}007.131.089.24$$

$$T = 27\ \text{days}\ 12^h\ 43^m\ 11^s{,}5 = 2.384.978{,}4958\ [\text{s}] \tag{14}$$

(T is the revolution period of Moon around the Earth),

$$1\ \text{average solar day} = 24^h\ 3^m\ 56^s{,}5554,$$

we have the major semiaxis of the ellipse described by the Moon around the Earth:

$$a = 384.767{,}780\ 375\ [\text{km}]. \tag{15}$$

Assuming the maximum distance of the Moon against Earth as being known:

$$a + c = 406.700\ [\text{km}] \tag{16}$$

the focal semidistance follows:

$$c = 21.932{,}219\ 625\ [\text{km}] \tag{17}$$

and thus we may calculate:

$$e = \frac{c}{a} = 0{,}057.001.185{,}5 \qquad \sqrt{a+c} = 637{,}730.350.54\ [\text{km}^{1/2}]$$

$$\sqrt{a-c}=602{,}358.332.514 \ [\mathrm{km}^{1/2}]$$

$$b=\sqrt{(a+c)(a-c)}=384.142{,}190\,544 \ [\mathrm{km}]$$

$$\frac{a}{b}=1{,}001.628.537.15 \qquad \left(\frac{a}{b}\right)^2=1{,}003.259.726.43 \tag{18}$$

$$\left(\frac{a}{b}\right)^4=1{,}006.530.078.67$$

$$\left(\frac{2\pi}{T}\right)^2=\left(\frac{62.831.853{,}07\cdot 10^{-7}}{2.384.978{,}4958}\right)^2=694{,}050.068.708\cdot 10^{-14} \ [s^{-2}]$$

Calculating in the points, A, B, C, D (or in any other point of the ellipse in fig. 2) the terms of eq. (1), we may easily see that formula :

$$C=\frac{|\bar{F}-m\dot{\bar{v}}|}{|[(m\bar{v}\times \mathrm{rot}\ \bar{v})\bar{r}_0]\bar{r}_0|} \tag{19}$$

maintains the same numerical value for C in every point of the ellipse as in the case of planets movement around the Sun, the constant C being thus a universal constant.

For instance, in point A, where

$$\theta=0,\ x=r=a+c,\ y=0, \tag{20}$$

$$m\dot{\bar{v}}_A=-\bar{i}m\left(\frac{2\pi}{T}\right)^2\left(\frac{a}{b}\right)^4\frac{(a-c)^2}{a}=-m\bar{v}_A\times \mathrm{rot}\ \bar{v}_A=$$

$$=-\bar{i}\,7{,}349\cdot 10^{22}\cdot 694{,}050.068.708\cdot 10^{-14}\cdot 1{,}006.530.078.67\cdot \tag{21}$$

$$\cdot\frac{362.835{,}56075^2}{384.767{,}780375}\cdot 10^3=-\bar{i}\,1{,}756.575.407.05\cdot 10^{20}$$

$$F_A=K\frac{Mm}{(a+c)^2}=6{,}664\cdot 10^{-11}\cdot\frac{5{,}975\cdot 10^{24}\cdot 7{,}349\cdot 10^{22}}{406{,}7^2\cdot 10^{12}}= \tag{22}$$

$$=1{,}769.101.703.09\cdot 10^{20}$$

$$C_A=\frac{|-1{,}756.575.407.05+1{,}769.101.703.09|\,10^{20}}{|1{,}756.575.407.05|\,10^{20}}=0{,}007.131.089.27 \tag{23}$$

For establishing the absolute movement of Moon in the solar system space, we must take into account that Earth trajectory around Sun is a three-dimensional curve resulted by compounding the translation movement to Vega star with a revolution movement around the Sun.

The revolution movement of the Earth around the Sun will be, for the Moon, a transport movement, which associated with the Moon revolution movement around the Earth as well as with the translation movement towards Vega star, will generate the absolute movement of the Moon throughout the solar system space.

STUDIUL MIŞCĂRII LUNII PLECÎND DE LA LEGEA GRAVITAŢIEI UNIVERSALE COMPLETATĂ CU FORŢA DE DEVIERE LATERALĂ

(Rezumat)

Mişcarea Lunii avînd la bază legea gravitaţiei universale, completată cu forţa de deviere laterală, reprezintă o altă confirmare a acestei legi, în care constanta *C* păstrează aceeaşi valoare, egală cu 0,077.131.089.27.

L'ÉTUDE DU MOUVEMENT DE LA LUNE AYANT À LA BASE LA LOI DE LA GRAVITATION UNIVERSELLE COMPLÉTÉE AVEC LA FORCE DE DÉVIATION LATÉRALE

(Résumé)

Le mouvement de la Lune ayant à la base la loi de la gravitation universelle, complétée avec la force de deviation latérale, représente une autre confirmation de cette loi, dans laquelle la constante *C* conserve la même valeur, égale à 0,007.131.089.27.

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C.Z.U. 662.753: 543.42.062

FUEL CONTAMINATION OF ENGINE OILS STUDIED THROUGH IR SPECTRAL ANALYSIS

BY

CONSTANTIN ŞUBRAN*

While being used engine oils get contaminated both by products of oxidation, polymerization, mechanical wear, etc. and by fuel, cooling fluid, dust particles. The latter factors bring about shortcomings which can influence the engine's normal lubrication. Their identification in used oils by means of spectral analysis offers the opportunity both for a rapid diagnosis of the engine's condition and for preventing defects or even engine breakage.

By rendering obvious an increased concentration of fuel (pet in oils one is warned about the segments abnormal condition, their leaîag and implicitly about power losses. Out of the oil-petrol mixture, through processes of cracking, pyrolysis and carbon-producing, there results a hard carbonous substance — calamine which subsides on the firing chamber walls, ignition plugs, valves, etc., thus causing a rapid wear that may end in segments failure. It has been considered that a higher than 1% concentration of petrol in oil becomes detrimental. Petrol contains molecules of light aromatic hydrocarbons with a single ring of the benzene and toluene kind.

The molecule of benzene has a high symmetry belonging to the D_{6h} symmetry group, in which all $C-C$ bonds are equivalent. Selection rules allow that, out of the twenty normal modes of vibration of C_6H_6 molecule, only four to be active in IR, a mode type A_{2u} and three of type E_{1u} (Fig. 1).

From a spectral viewpoint petrol contamination of oil becomes manifest especially by the appearance of bands at 700—800 cm^{-1}, a range exclusively occupied by aromates and substituted derivatives of benzene. These bands are due to γ_{CH} in-phase deformations of H atoms which are perpendicular to the ring surface (medusa movement). The substitution of one or more hydrogen atoms leads to a symmetry weakening. The mono substituted molecule will be included in the symmetry group C_{2v}; the number of modes of vibration active in IR will be much larger. Inphase bending vibrations γ_{CH} give rise to intense IR bands, characteristic in the

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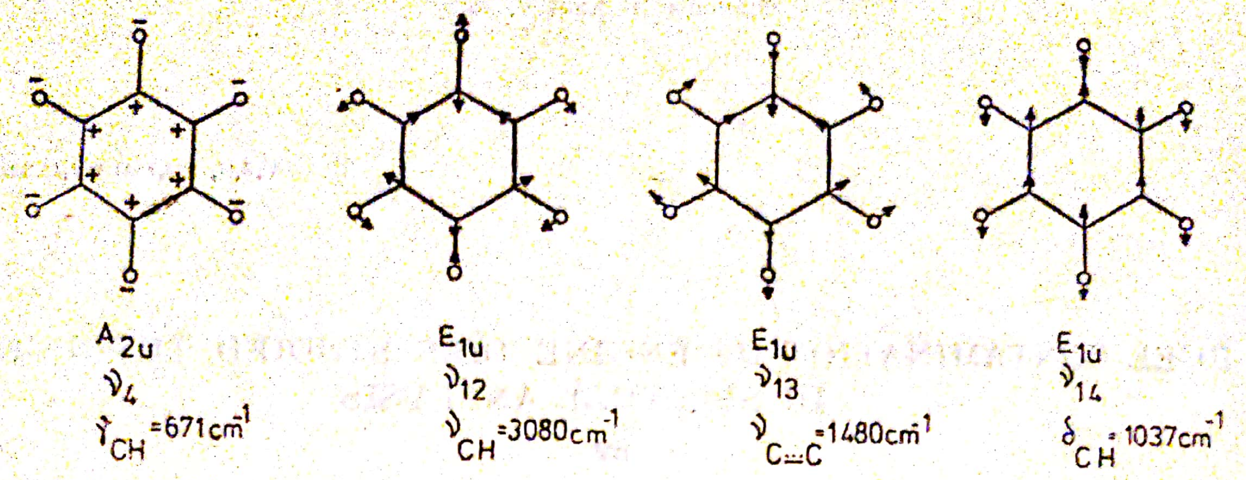


Fig. 1

range 680—900 cm- ¹ because *C* and *H* atoms vibrate in opposite sense and cause some fairly important changes of the dipole moment (Fig. 2).

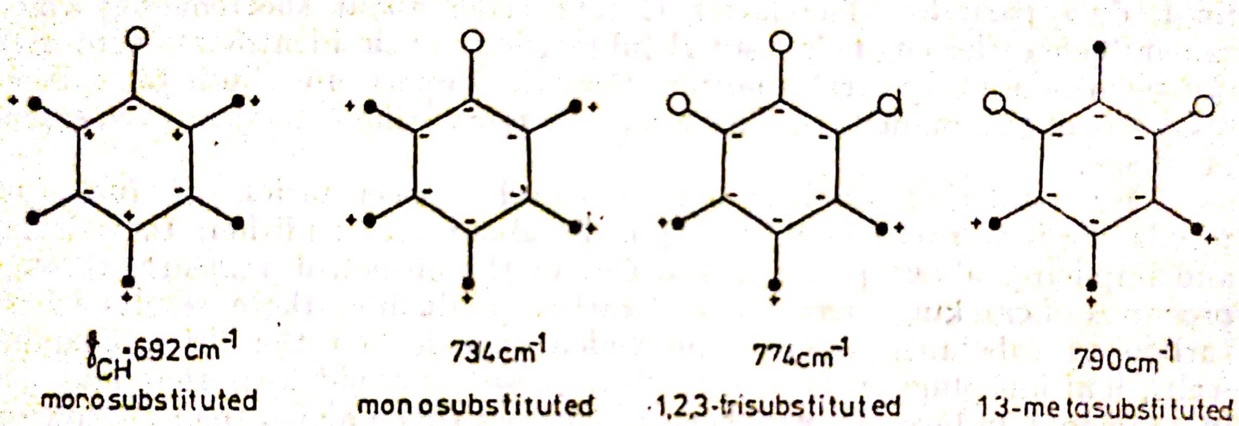
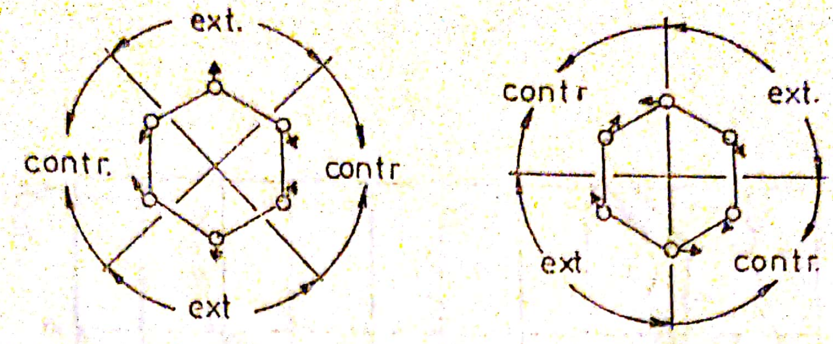


Fig. 2

In the case of substituted derivatives the bands appear towards higher frequencies which depend only on the substituents number an position and not on their nature. There is also a correlation between substitution degree increase and the bands shifting towards higher frequencies. Due to changes in the molecule's symmetry other weaker bands may also appear in the 680—900 cm^{-1} range; they are caused by certain out-of-plan deformations which become active in *IR*.

The spectrum in Fig. 4 shows the pure oil spectrum, the 4% petrol contaminated oil, and the differential spectrum of the two samples in0.06 mm cells. One can immediately see the appearance of peaks due to petrol in the 680—900 cm^{-1} range. The confirmation comes from the band at 1 595 cm^{-1}

This band is due to some carbon-carbon vibrations, that is, some skelet al ring breathing modes like those shown in Fig. 3. All the other bands due to the possible ν_{CH}, δ_{CH}, ρ_{CH}, ω_{CH}, ν_{C-C} modes of vibration do not have analytical value because of the numerous interferences with other bands due to the complex structure of mineral oils.



$$\nu_{C=C} = 1595\ cm^{-1}$$

Fig. 3

For an accurate representation the spectrum in Fig. 5 has been drawn; it has an extension on the abscissa in the 1 200—660 cm^{-1} range. There can be noticed the appearance of peaks at 692, 734, 758, 774, 796 cm^{-1} whose significance has been indicated in the figure. As this range is not affected by other bands it can be most useful in detecting the petrol content in engine oils. The appearance of these peaks (even at a 3% concentration) constitutes the reference mark for a petrol excess in oils.

IR measurements and analyses have been made by means of a double beam spectrophotometer SPECORD 75 *IR*. Dispersion of radiation is achieved with the help of a grating and a *KBr* prism which separates spectral orders of grating *R*. Contaminated oil samples have been analysed in layers of 0.10 mm, by using demountable cells with *KBr* windows.

Spectra have been traced both by emplying the *IR* classical method and by differential *IR* spectrometry. In the case of the latter the measure cells contained the contaminated oil sample while the reference cell held the original, new oil sample. Any change occurring in the structure of contaminated oil by comparison with new oil is recordered by a series of distinct spectral bands; down — oriented bands acknowledge the occurrance either of new compounds or of contamination itself.

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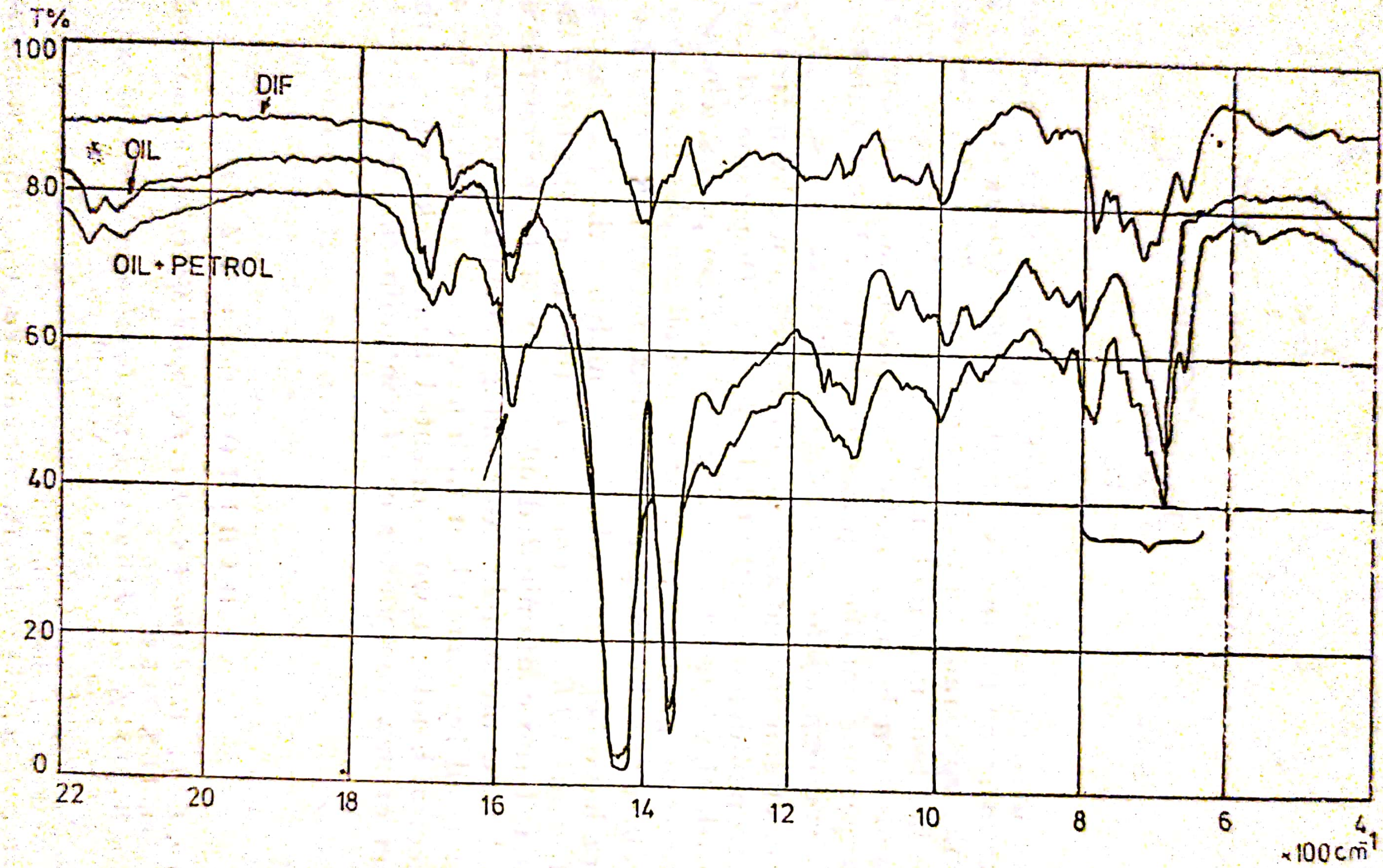
T%
100
80
60
40
20
0
DIF
OIL
OIL+PETROL
22
20
18
16
14
12
10
8
6
4
×100 cm⁻¹

Fig. 4

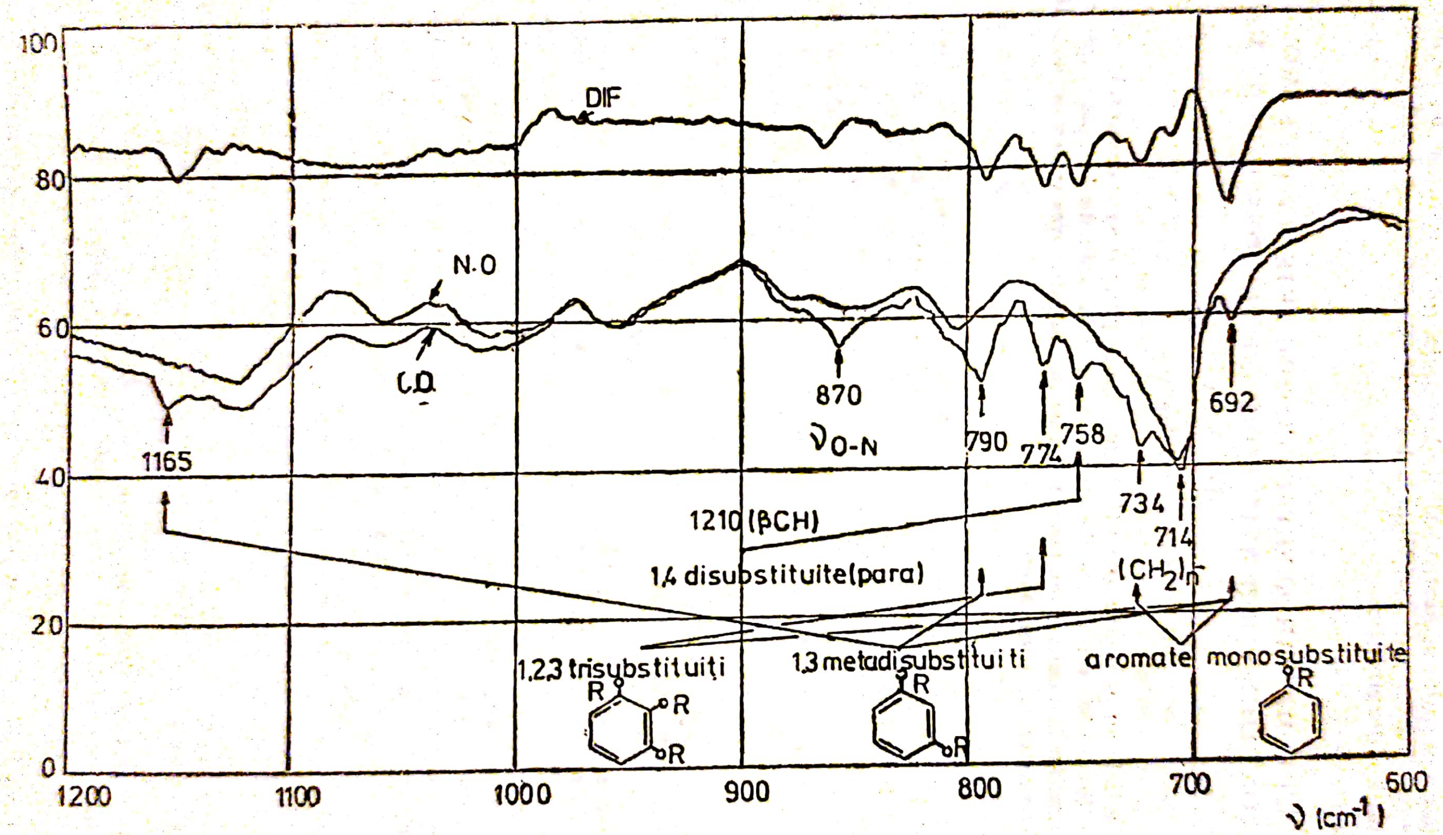
100
80
60
40
20
0
DIF
N.O
C.O
870
ν_{O-N}
1165
790
774
758
734
714
692
1210 (βCH)
1,4 disubstituite(para)
$(CH_2)_n$
1,2,3 trisubstituiţi
1,3 metadisubstituiti
aromate monosubstituite
R
1200
1100
1000
900
800
700
600
ν (cm^{-1})

Fig. 5

STUDIUL PRIN SPECTROFOTOMETRIE ÎN INFRAROȘU A CONTAMINĂRII CU COMBUSTIBIL A ULEIURILOR MINERALE DE MOTOR

(Rezumat)

În articol se prezintă o metodă de diagnosticare rapidă a contaminării excesive cu benzină a uleiurilor minerale de motor utilizînd metoda spectrofotometriei în infraroșu. Identificarea benzilor de absorbție și poziționarea lor se realizează cu ajutorul teoriei grupurilor și simetriei modurilor de vibrație.

L'ÉTUDE PAR SPECTROPHOTOMÉTRIE EN INFRAROUGE DE LA CONTAMINATION AVEC COMBUSTIBLE DES HUILES MINÉRALES DE MOTEURS

(Résumé)

Dans l'article on présente une méthode rapide pour diagnostiquer la présence excessive d'essence dans les huiles minérales des moteurs par la voie de la spectrophotométrie en infrarouge. On justifie la position et les intensités des bandes spectrales en coordination avec la symmétrie des modes de vibration.

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D. MANGERON, N. IRIMICIUC: *Mécanique des rigides ayant des applications dans l'ingéniérie* (3-ème volume): *Mécanique des vibrations des systèmes de rigides* (en roumain); Editura tehnică, Bucarest, 1981 550 p.

La *Mécanique des vibrations des systèmes de rigides* représente le dernier volume du traité conçu en 3 (trois) volumes: *Mécanique des rigides ayant des applications dans l'ingéniérie*, les deux autres volumes étant parus dans la même maison éditrice en 1978, respectivement 1980*.

Ce volume, qui forme la 5-ème partie du traité mentionné, est systématisé sur 3 (trois) chapitres numérotés à la succession des volumes précédents, comme suit:

Chap. IX: Cinématique des vibrations des systèmes de rigides;
Chap. X: Dynamique des vibrations des systèmes de rigides;
Chap. XI: Stabilité des mouvements des systèmes vibrants.

L'ensemble des problèmes abordés représente la partie la plus récente de la mécanique des systèmes de rigides, partie qui a également, les applications actuelles les plus diverses. En même temps, cette partie est la plus complexe et la plus difficile à systématiser dans le but d'une présentation unitaire.

Dans le chap. IX, après qu'on définisse, d'abord, les notions fondamentales utilisées dans la théorie et la pratique des vibrations, on fait la classification des mouvements vibratoires-selon le criterium cinématique—, on établi les formes des équations différentielles pour les vibrations de type déterministe et on présente des cas divers concernant la composition des vibrations.

Dans le chap. X sont présentées, dans la première partie, les caractéristiques généralisées (inertiales, cinématiques et dynamiques) des systèmes vibrants; puis, on analyse: l'excitation des systèmes vibrants, les formes des équations différentielles des mouvements vibratoires des systèmes mécaniques et la réponse des systèmes vibrants soumis aux excitations déterministes et aléatoires.

Dans le chap. XI, pour les systèmes vibrants autonomes et non autonomes, on fait l'étude de la stabilité en première approximation.

La présentation dans un espace tellement restreint — le livre ne compte que 547 pages — d'un si vaste matériel a été possible, d'un coté, grâce à l'adoption de l'idée de la modélation mathématique des vibrations mécaniques et, de l'autre coté, par la sélection très soignée des cas qu'on expose, cas qui illustrent le mieux le phénomène étudié et présentent, en même temps, le plus grand intérêt du point de vue technique.

Compte tenu de l'importance des applications techniques des systèmes de rigides en vibration, dans beaucoup de cas — comme l'est, notament, celui des excitateurs — les auteurs ne se bornent pas seulement à une étude théorique des phénomènes, mais, sur la base des conclusions tirées de leur étude théorique, ils formulent les principes pratiques sur lesquels doit reposer la conception des dispositifs dont le fonctionnement est basé sur ces phénomènes.

Cette façon d'aborder simultanément les aspects théoriques et pratiques (surtout dans les conditions actuelles du développement de la technique, quand les vitesses et les masses en mouvement, des machines, augmentent incessamment) est de nature à offrir aux chercheurs des directions nouvelles d'investigation dans le domaine de la dynamique des machines et à inspirer aux ingénieurs des solutions techniques d'une fiabilité à envier.

La présentation est claire, précise et concise — qualités qu'on a remarquées aussi dans les autres deux volumes; on utilise toujours l'appareil mathématique adéquat, l'angagement pris dans la préface du premier volume, de „bannir tout spéctacle mathématique stérile" „étant strictement respecté".

Par l'apparition de ce dernier volume, le traité—trilogie, *Mécanique des rigides ayants des applications dans l'ingéniérie*, couvre un très vaste ensemble des aspects théoriques et pratiques de la mécanique du rigide et des systèmes de rigides.

Ce traité, par la richesse des sujets abordés, par l'accessibilité de l'exposé, ainsi que par l'unité de l'ensemble, est un ouvrage de référence qui, sans aucune doute, fera date dans son domaine.

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* Voir: 1. Revue roumaine des sciences techniques, série de *Mécanique appliquée*, tome 26, nr. 6, novembre—décembre, pag. 921, Bucarest—Roumanie, 1981;
2. Buletinul Universităţii din Galaţi, Fascicola II, Matematică, fizică, mecanică teoretică, Anul IV, pag. 93—94, Galaţi—România, 1981.

Notre conclusion est renforcée par nombre de comptes rendus de haute teneur parus à l'étrager concernant la même trilogie. Bornons nous d'en citer les dernières trois phrases du compte rendu relatif au second volume de cette trilogie, inséré, sous la signature autorisée du Prof. Dr. V. M., Starzhinski, titulaire de l'une des chaires de Mécanique de l'Université „*M. V. Lomonosov*" de Moscou, dans le périodique spécialisé *Novye knigi za rubezhom*, Série A, fasc. 2, 1982, pp. 42–43, à savoir :

„En évaluant les deux premiers volumes de la trilogie, il faut souligner l'originalité de sa construction et l'exposition, de même que sa saturation par différentes applications. Tout ça fait de cette trilogie non seulement un manual, mais aussi un précieux traité de référence. Il faut recommander, avec insistance, la traduction de cette trilogie en russe et l'édition, chez nous de celle-ci".

ION GRUDU et GHEORGHE MOMANU
L'Université de Galați (Roumanie)

M. VUKOBRATOVIĆ, V. POTKONJAK, *Dynamics of Manipulation Robots. Theory and Application. Scientific Fundamentals of Robotics.* I. Communications and Control Engineering Series. Springer-Verlag, Berlin, Heidelberg—New York, XIII, 303 p., 149 fig., 1982.

Parallélement à toute une série de travaux publiés par les représentants des puissantes écoles d'Allemagne, du Japon, de l'U.R.S.S. et de l'U.S.A., suivi par nombre de résultats et de réalisations remarquables, de publications, de conférences périodiques et de livres élaborés dans quelques-uns des pays qui font des efforts remarquables de développement accéléré, tel, par ex., la Roumanie, concernant l'ensemble des études et des techniques tendant à concevoir des systèmes capables de substituer à l'homme dans ses fonctions motrices, sensorielles et intellectuelles, qui répondent au terme *robotique*, on assiste de nos jours à l'ensemble des efforts qui se réfèrent à l'élaboration et à la construction effective des appareils capables d'agir de façon automatique pour une ou plusieurs fonctions données, qui répondent au terme *robot*. On doit souligner dans cet égard l'excellente contribution portant tant à la robotique quant aux robots due à l'Institut „MIHAILO PUPIN" de Belgrade, qui a suscitée, entre autres, une résonance constructive et profonde parmi les représentants des écoles roumaines de Mécanique et de Théorie des mécanismes et des machines et a conduit d'or et déjà non seulement à l'organisation périodique des conférences sur robotique et robots et des symposiums de Théorie des mécanismes et des machines grâce à l'enthousiasme et compétence bien difficile à surestimer de ses organisateurs, à savoir MM. les professeurs Christian Pelecudi et Nicolae Manolescu, mais aussi à l'élaboration des livres sur robotique et robots déjà publiés ou bien en cours de publication dûs, d'après ce que nous savons, à M. le Prof. et Doyen Francisc Kovács et à M. le Prof. Nicolae Ioniță et à la constructions effective d'un nombre de robots à Timișoara, Iași, Bucarest et Cluj-Napoca.

Le plus actif des chercheurs de l'Institut „MIHAILO PUPIN" est sans aucun doute le premier des auteurs du livre dont le compte rendu s'ensuit et qui s'encadre dans la série „*Communications and Control Engineering Series*", dont les éditeurs sont MM. A. Fettweis, J. L. Massey et M. Thoma et ouvre en même temps la série plus restreinte, à savoir „*Scientific Fundamentals of Robotics*". Dans le premier chapitre (pp. 1—25), intitulé „Remarques générales concernant robots et dynamique des manipulateurs", il s'agit des mécanismes spatiaux actifs et des méthodes analytiques de formation des équations dynamiques concernant des corps rigides rejoints, actifs et spatiaux. Bibl. : 24 titres, dont deux sont dûs au premier des auteurs et les trois autres sont en collaboration avec D. Stokić. Les lecteurs qui suivent régulièrement les comptes rendus des travaux dans cet ordre d'idées qui s'insèrent dans les périodiques de bibliographie critique APPLIED MECHANICS REVIEWS, REFERATIVNYI ZHURNAL MEKHANIKA ET ZENTRALBLATT FÜR MATHEMATIK UND IHRE GRENZGEBIETE ont lu sans doute les renvois des plus récents concernant les Nos 3 et 4 de la Bibliographie. Il s'agit des travaux dûs à M. M. Vukobratović et D. Stokić. Le chapitre 2 (pp. 26—149), tout en représentant la partie centrale de cette monographie et se référant à la robotique, intitulé „Méthodes qui utilisent des ordinateurs pour l'établissement et résolution des modèles mathématiques des mécanismes actifs", est divisé en trois parties, à savoir : „Méthodes basées sur les théorèmes généraux de dynamique et les équations de Newton-Euler" (pp. 31—87), „Méthodes basées sur les équations de Lagrange" (pp. 87—116) et „Méthodes basées sur le principe de Gauss et les équations d'Appell" (pp. 116—149). On y trouve crystallisée pour la première fois une vision complète sur les méthodes existentes que l'on utilise pour la formation automatique des modèles mathématiques de la dynamique des chaînes cinématiques, ayant des configurations spatiales arbitraires. Bibl. : 23 titres, dont quatre sont dûs au premier des auteurs et quatre autres à tous les deux, et, enfin, cinq autres au premier des auteurs en collaboration avec d'autres chercheurs. Les ré-

férents soulignent surtout les sujets exposés dans les pages 87—116 qui se réfèrent aux travaux les plus récents des auteurs marqués par les Nos 14 de la Bibliographie. Le chap. 3 : „Simulation de la dynamique des manipulateurs et réglage vers des mouvements fonctionnels“ (pp. 150—218) se réfère aux applications des méthodes exposées dans le Chap. précédent ayant pour but la synthèse des mouvements fonctionnels dans les cas des tâches typiques des manipulateurs, illustrés surtout dans les figures 3.4 (p. 161) et 3,36 (p. 202). On y trouve élaborées non seulement différentes chaînes fonctionnelles aptes à résoudre par nombre de voies des problèmes où l'on utilise des manipulateurs de différents degrés de mobilité, mais aussi d'intéressants examples de synthèse des mouvements fonctionnels des robots de manipulation de différentes configurations. Bibl. : 30 titres, dont quelques-uns ont été déjà mentionnés auparavant et dont trois encore sont dûs au premier des auteurs, six à tous les deux, et dix au premier en collaboration avec d'autres chercheurs. Le chap. 4 : „Dynamique des manipulateurs aux éléments élastiques“ (pp. 219—251) contient le développement de l'analyse micro-dynamique, dont le modèle mathématique est donné sous la forme (4.2.2, p. 222) et représenté dans la Fig. 4.3, p. 225. C'est une méthode simplifiée pour le calcul des oscillations élastiques. Bibl. : 28 titres. On y trouve entre autres la reproduction partielle des deux travaus des auteurs, encore plus récents, marqués dans la Bibl. par les Nos 27 et 28, insérés dans le Journal de l'IFToMM : MECHANISM AND MACHINE THEORY. Cet excellent volume, où la rigurosité des déductions mathématiques et applications industrielles illustrées par de nombreuses figures sont liées d'une manière harmonieuse bien difficile à surestimer, conclut par le Chap. 5 : „Méthode dynamique pour l'évaluation et choix des manipulateurs industriels“ (pp. 252—299). On y trouve l'exposition d'une méthode à l'aide des ordinateurs apte à l'élaboration des avant-projets des robots de manipulation, tout en tenant compte de différents critères et de constraints imposés, doués de performances dynamiques désirées et de caractéristiques optimales. Bibl. : 25 titres, dont le plus récent, No. 19, dû à tous les deux, pas encore analysé dans les pages des périodiques internationaux de bibliographie critique mentionnés ci-dessus, est inséré dans le Journal de l'IFToMM „Mech. and Mach. Theory“, No. 3, 1982, pp. 300—303, Vol. 17, Faut-il mentionner aussi la présence à la fin du volume d'un Index des sujets très bien choisis et ordonnés.

Somme toute, les référents de cette Monographie, qui offre à lecteur une base très solide pour l'étude avancée de la dynamique des robots, élaborée surtout grâce à un large nombre de travaux dûs aux auteurs tant qu'à quelques autres membres de l'Institut „Mihailo Pupin“ (Yougoslavie), tout en appropriant la très modeste assertion du premier des auteurs, extraite de sa lettre très récente adressée à l'un de nous, que les „complete results from dynamics of open active mechanisms in a cleaned-up and correct form are given in the Monograph“ et tout en soulignant encore une fois les mérites de cet excellent oeuvre, en sont sûrs que ce volume et le suivant : „Scientific Fundamentals of Robotics. 2. Control of Manipulation Robots. Theory and Application“, dûs aux M. Vukobratović et D. Stokić, édités par les soins à admirer du Springer-Verlag, Berlin—Heidelberg—New York, ne tarderont pas d'être traduits dans d'autres langues de nôtre Planète.

D. Mangeron, V. Giurcă, Aurora Crăciunaș, Institut Polytechnique „Gheorghe Asachi“ de Iași ; S. T. Chiriacescu, Université de Brașov ; I. Crudu et Gh. Momanu, Université de Galați et I. B. Bălan, Institut Polytechnique de Cluj-Napoca, République Socialiste de Roumanie.

M. VUKOBRATOVIĆ, D. STOKIČ, *Control of manipulation robots. Theory and application. Scientific Fundamentals of Robotics*, 2. Communications and Control Engineering Series, Springer-Verlag, Berlin-Heidelberg-New York, XIII, 366 p., 111 figures, 1982.

Ce volume, dont l'apport théorique et pratique, ayant pour but l'approfondissement ultérieur de la robotique et la réalisation effective des robots à tout fins largement doués de l'intelligence artificielle remarquable, est bien difficile à surestimer, s'encadre dans la série „COMMUNICATIONS AND CONTROL ENGINEERING SERIES, dont les éditeurs sont MM. A. Fettweis, J. L. Massey et M. Thoma, et en est subséquent au premier volume, élaboré par MM. M. Vukobratović et V. Potkonjak de l'Institut „MIHAILO PUPIN“ de Belgrade, fait partie, à son tour, de la série SCIENTIFIC FUNDAMENTALS OF ROBOTICS, à savoir : „Dynamics of Manipulation Robots. Theory and Application“.

Ayant à la base une nouvelle approche à ce que l'on appelle *contrôle suboptimal des systèmes dynamiques à large échelle* et en particulier celui qui se réfère à la robotique, cette excellente Monographie comporte une partie introductive consacrée à l'exposition des principes généraux concernant la synthèse du contrôle des robots et manipulateurs (pp. 1—30. Bibl. : 71 titres, dont 3 appartiennent au premier des auteurs, 9 à tous les deux et, enfin, 10 autres sont en collaboration du premier des auteurs ou bien de tout les deux avec d'autres chercheurs), trois chapitres et quatre appendices.

Dans le premier chapitre, qui en assure l'autonomie de ce volume en ce que regarde les modèles mathématiques de la classe des systèmes prise en considération, on trouve l'exposition d'une procédure, basée sur les théorèmes généraux de la Mécanique, pour la construction, aidée par ordinateur, qui conduit à l'établissement des modèles mathématiques de la dynamique des mécanismes spatiaux actifs (pp. 48—57), tant que d'une autre procédure, réalisée toujours à l'aide des calculatrices électroniques, pour la linéarisation des modèles dynamiques des chaînes cinématiques ouvertes (pp. 67—68 sont consacrées à une Bibliographie de 18 titres, dont huit sont dûs aux auteurs seuls ou bien en collaboration avec d'autres chercheurs et dont le No. 1 se réfère au premier volume de la série restrainte citée ci-dessus).

Le deuxième chapitre (pp. 69—166), ayant à la fin les appendices : a.A. „Suboptimality of Global Control" (pp. 151—152) ; 2.B. „Analysis of „Distribution of the Model" between Subsystems and Coupling" (pp. 153—156) et 2.C. „Stability Analysis of System with Decentralized Regulator and Observer" (pp. 157—166), expose une procédure générale pour la synthèse du contrôle des systèmes mécaniques à large échelle, basée sur la synthèse des phases du contrôle global et décentralisé de la dynamique perturbée (pp. 101—126), suivie par celle du contrôle à temps discretisé qui en assure l'utilisation des microordinateurs et la construction d'un algorithme pour des calculatrices électroniques numériques afin de pouvoir parvenir à la synthèse d'un contrôle relativement simple se référant tant aux configurations des manipulateurs arbitraires, quant au contrôle des différentes tâches (pp. 138—145). On y trouve en particulier pour des systèmes manipulés la synthèse du régulateur linéaire optimal centralisé classique. La Bibliographie de 47 titres (pp. 147—150), dont 20 travaux sont dûs au premier des auteurs ou bien à tout les deux, parfois en collaboration avec d'autres chercheurs, est précédée par une conclusion, où l'on trouve entre autres précisés quelques-uns des problèmes que l'on résout dans le chap. suivant.

Dans le chapitre 3, intitulé „Control Synthesis for typical Manipulations Tasks", on trouve analysés en détail, tout en utilisant les modèles mathématiques des manipulateurs et la construction effective de ceux-ci à l'aide des calculatrices électroniques numériques, exposés dans la chap. I et tout en puisant des exposés et des procédures des synthèses du contrôle présentés dans le chap. 2, quelques exemples des tâches que l'on rencontre fréquemment dans l'industrie. Telles tâches sont sous-divisées en trois catégories, à savoir : 1) Transfer de l'extremité d'un manipulateur (pince à serrer et objet à ouvrager) le long d'une trajectoire prescrite dans l'espace (pp. 176—221) ; 2) Transfer de l'objet à ouvrager en y conservant durant le transfer l'orientation désirée de celui-ci le long d'une trajectoire prescrite dans l'espace et contrôle de l'orientation du pince à serrer (pp. 222—297) et 3) Processus d'insertion d'un objet dans un orifice fixé en utilisant la marche arrière de la charge, qui en constitue la phase principale (assez délicate) d'assemblage par un manipulateur (pp. 297—320).

Tout en excluant les tâches qui reclament l'abordage systématique du problème de l'intelligence artificielle, les résultats de la synthèse du contrôle des tâches ci-dessus se réfèrent à trois types de manipulateurs industriels, à savoir : manipulateurs de type anthropomorphique ; manipulateurs ayant degrés de liberté linéaires et manipulateurs semi-anthropomorphiques, conçus et construits dans le cadre de l'Institut „MIHAILO PUPIN" de Belgrade.

La Conclusion (pp. 336—337) pointe les sortes des contrôles exécutés tant que la possibilité de l'exécution de quelques-unes autres structures de contrôle et souligne l'exclusion de ce volume des problèmes concernant contrôle adaptif, non linéaire etc. La Bibliographie (pp. 337—340), qui contient 36 titres, dont 25 appartiennent aux auteurs, parfois en collaboration avec d'autres chercheurs, et au moins six autres sont dûs aux membres de ce même bien fameux Institut „MIHAILO PUPIN", est suivie par l'appendice 3.A. où se trouve exposé un algorithme de simulation se référant à l'assemblage (pp. 341—360). Faut-il mentionner encore l'excellence des figures, de l'index des matières, du choix des méthodes mathématiques rigoureuses des plus efficaces, qui, les référants en sont sûrs, seront très appreciées tant par les théoriciens que par les praticiens (ingénieurs et techniciens), bien que le fait de sa parution dans les conditions hors concours chez Springer-Verlag.

Somme toute ce volume, tout en contribuant pleinement à l'échafaudage de la nouvelle discipline spécifique de nôtre age, que l'on appelle *robotique*, réunit d'une manière très heureuse les connaissances multidisciplinaires de niveau élevé et les résultats dûs à la clairvoyance et à la faculté d'invention à admirer des auteurs et constitue avec le premier volume de la série déjà citée un chef-d'oeuvre que nous tous en devons posséder, apprendre et utiliser dans le cadre de la créativité, tandis que nombre d'autres spécialistent trouveront, s'il en sera besoin, ce chef-d'oeuvre traduit sous peu dans quelques-unes d'autres langues de nôtre Terre.

D. Mangeron, V. Giurcă, Aurora Crăciunaş, Institut Polytechnique „Gheorghe Asachi" de Iaşi ; S. T. Chiriacescu, Université de Braşov ; I. Crudu, Gh. Momanu, Université de Galaţi ; I. B. Bălan. Institut Polytechnique de Cluj-Napoca et N. Ioniţă, Académie des Études Économiques, Bucarest, République Socialiste de Roumanie.

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Tomul XXXIII, s. I a. Matematică, 1987, f. 2.

LE PROBLÈME DES DEUX CORPS DANS UN REPÈRE NON GALILÉEN. L'AVANCE DE PÉRIHÉLIE. LE MOUVEMENT DES COMÈTES

PAR

C. I. BORȘ

L'Univers dont on s'occupe est formé par deux points matériels S et P qui s'attirent d'après la loi de Newton.

Par rapport à un repère galiléen $\mathcal{S}_0$, les équations de mouvement sont données par

$$m_1\ddot{\mathbf{r}}_1 = f\frac{m_1m_2}{\mathbf{r}^2}\,\mathbf{u}, \quad m_2\ddot{\mathbf{r}}_2 = -f\frac{m_1m_2}{\mathbf{r}^2}\,\mathbf{u}, \tag{1}$$

où m_1, m_2 sont les masses de deux points, $\mathbf{r}_1$, $\mathbf{r}_2$ leur positions dans $\mathcal{S}_0$ et $\mathbf{u} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{\mathbf{r}}{r}$ est le vecteur unité de $\mathbf{r} = \overrightarrow{SP}$.

Le centre d'inertie G du système S et P a, par rapport à $\mathcal{S}_0$, un mouvement rectiligne et uniforme. Il exist donc un repère galiléen $\mathcal{S}^*$ ayant l'origine au centre d'inertie G. Dans ce nouveau repère les forces qui agissent sur les deux points sont centrales et, par conséquent, les trajectoires décrites par les deux points seront situées dans un plan qui est fixe dans $\mathcal{S}^*$. Il suit que la normale au plan des trajectoires a une direction fixe.

Considérons maintenant un système d'axes de coordonnées triorthogonal $Sx_1x_2x_3$ ayant l'origine en S et dont le plan Sx_1x_2 reste en coïncidence avec le plan des trajectoires de $\mathcal{S}^*$. Le repère $Sx_1x_2x_3$ n'est pas galiléen. L'accélération de S par rapport à $\mathcal{S}_0$ est donnée par la première équation (1). La rotation du repère $Sx_1x_2x_3$ par rapport à $\mathcal{S}^*$ est de la forme $\mathbf{w} = \omega\mathbf{e}_3$ où $\mathbf{e}_3$ est le vecteur unité de la direction fixe de Sx_3.

Le mouvement de P dans le repère $Sx_1x_2x_3$ est décrit par l'équation

$$m_2\ddot{\mathbf{r}} = -f\frac{m_1m_2}{\mathbf{r}^2}\,\mathbf{u} - m_2\left[f\frac{m_2}{\mathbf{r}^2}\,\mathbf{u} + \dot{\mathbf{w}}\times\mathbf{r} + \mathbf{w}\times(\mathbf{w}\times\mathbf{r}) + 2\mathbf{w}\times\mathbf{v}\right]. \tag{2}$$

où $\mathbf{r}$ et $\mathbf{v}$ représentent la position et la vitesse de P par rapport à $Sx_1x_2x_3$.

La trajectoire sera dans le plan Sx_1x_2. Dans un système polaire de coordonnées r, θ, l'angle θ étant mesuré à partir de l'axe mobile Sx_1, nous obtenons les équations

$$\begin{aligned} \ddot{r} - \mathbf{r}\dot{\theta}^2 &= -f\frac{m_1+m_2}{r^2} + \omega^2 r + 2r\omega\dot{\theta}, \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= -r\dot{\omega} - 2\dot{r}\omega. \end{aligned} \tag{3}$$

Le repère polaire **u**, **n**, $\mathbf{e}_3$ est choisi tel que $\mathbf{un}=0$, $\mathbf{ne}_3=0$, $\mathbf{e}_3\mathbf{u}=0$, $(\mathbf{u}, \mathbf{n}, \mathbf{e}_3)=1$.

La seconde équation du système (3) nous fournit l'intégrale première

$$r^2(\dot{\theta}+\omega)=C \quad \text{(const.)}. \tag{4}$$

En multipliant $(3)_1$ par $\dot{r}$ et en intégrant, on peut déduire encore une intégrale première du mouvement (l'intégrale de Painlevé) donnée par

$$\frac{1}{2}\dot{r}^2=f\frac{m_1+m_2}{r}-\frac{1}{2}\frac{C^2}{r^2}+h=\frac{Q_1(r)}{2r^2}, \tag{5}$$

$$Q_1(r)=2hr^2+2f(m_1+m_2)r-C^2, \tag{6}$$

où h est une constante qui est déterminée par les données initialles. L'intégrale première (5) a la même forme que dans le cas classique, à la seule différence près que C a une interprétation modifiée. Cette intégrale première peut être obtenue aussi en appliquant le théorème de l'énergie. La nature de la trajectoire dépendra du signe de h.

De (5), on peut obtenir l'équation différentielle de la trajectoire qui donne r comme fonction de θ. On peut établir aussi une formule de type Binet.

En tenant compte que d'après des observations astronomiques récentes, le repère de Copernic n'a pas les axes de directions fixes (le repère ayant l'origine au centre du Soleil et les directions des axes dirigées vers trois étoiles supposées fixes) on peut appliquer les résultats précédents pour décrire le mouvement d'une planète autour du Soleil.

Supposons d'abord que le repère $Sx_1x_2x_3$ a, par rapport à $\mathcal{S}^*$, un mouvement spécial tel que

$$r^2\omega=c_1 \quad \text{(const.)}. \tag{7}$$

Alors, en utilisant (4), on déduit

$$r^2\dot{\theta}=c_2 \quad \text{(const.)}, \tag{8}$$

c'est-à-dire la loi des aires est vraie dans le repère $Sx_1x_2x_3$.

Grâce à ces résultats la première équation (3) devient

$$\ddot{r}-r\dot{\theta}^2=-f\frac{m_1+m_2}{r^2}+\frac{c_1(c_1+2c_2)}{r^3}. \tag{9}$$

Le second terme du membre droit de l'équation précédente a la même forme que le terme introduit par Clairaut pour expliquer le mouvement de la Lune et par Maillard pour justifier les avances des périhélies planétaires (Pour des renseignements d'ordre historique voir [2], [3], [4]).

D'après ce qui précède, l'existence d'un tel terme est une conséquence des lois de la mécanique newtonienne dans un repère non galiléen et, par conséquent, l'introduction d'un tel terme ne signifie pas „la correction" de la loi d'attraction universelle de Newton. De cette manière, les avances des périhélies des planètes s'expliquent dans le cadre de la mécanique newtonienne.

Maintenant, si l'on renonce à l'hypothèse (7) la première équation (3), en vertu de (4), devient

$$\ddot{r}-r\dot{\theta}^2=-f\frac{m_1+m_2}{r^2}+\frac{C^2}{r^3}-r\left(\frac{C}{r^2}-\omega\right)^2, \tag{10}$$

et les conclusions concernant les avances des périhélies planétaires seront les mêmes.

Dans ce qui suit, nous allons nous occuper plus en détail de la force totale **F** qui agit sur P (dans le repère $Sx_1x_2x_3$).

En vertu de (3) nous pouvons écrire :

$$\mathbf{F}=m_2\left(-f\frac{m_1+m_2}{r^2}+\omega^2r+2r\omega\dot{\theta}\right)\mathbf{u}-m_2(\dot{\omega}r+2\omega\dot{r})\mathbf{n}.$$

En tenant compte de l'intégrale première (4), on trouve

$$\begin{aligned}\mathbf{F}&=m_2\left(-f\frac{m_1+m_2}{r^2}+\frac{2C\omega}{r}-\omega^2r\right)\mathbf{u}-m_2(\dot{\omega}r+2\omega\dot{r})\mathbf{n}=\\&=\frac{m_2Q_2(r)}{r^2}\mathbf{u}-m_2(\dot{\omega}r+2\omega\dot{r})\mathbf{n},\end{aligned} \tag{11}$$

où

$$Q_2(r)=-\omega^2r^3+2C\omega r-f(m_1+m_2). \tag{12}$$

D'après ce que l'on voit, la force totale **F** se compose d'une force centrale et d'une force transversale. La force centrale a trois composantes : l'attraction newtonienne, une force qui est inversement proportionnelle à la distance (de répulsion lorsque $C\omega>0$ et d'attraction si $C\omega<0$) et une force élastique.

Le sens de la force centrale (de répulsion ou d'attraction) est donné par le signe du polynôme $Q_2(r)$. Le polynôme (12) a une racine négative. Si les deux autres racines r_1 et r_2 ($r_1\neq r_2$) du polynôme (12) sont réelles alors elles sont positives; elles sont séparées par le nombre $d_1=(2/3\,C/\omega)^{1/2}$, $C\omega>0$ et nous aurons $Q_2(d_1)>0$ et $Q_2(r)\geqslant 0$ pour $r_1\leqslant r\leqslant r_2$.

On aura donc une couronne circulaire autour de S définie par $r_1<r<r_2$ où la force centrale est répulsive.

D'après ce que l'on voit, étant donné un mouvement du point P autour de S, la constate C est celle qui indique si nous pouvons avoir une couronne autour de S où la force centrale qui agit sur P soit répulsive. Si la trajectoire de P traverse ou non cette couronne ceci dépend de la constante h de (5), ce qui peut être déterminé par les positions des racines des deux polynômes $Q_1(r)$ et $Q_2(r)$.

Si le point P est l'image d'une comète qui se déplace dans le système solaire et pénètre dans sa couronne de répulsion, alors, pendant le mouvement à travers cette région, peut se former la queue de la comète. Si la comète traverse la circonférence de rayon r_1 (surtout si la comète a la trajectoire tout au long de cette circonférence pendant une période plus longue), elle peut se diviser. Si, après avoir passée par la couronne de répulsion, la

comète a une portion de trajectoire dans l'intérieur du cercle de rayon r_1, une seconde queue peut se former, celle-ci dirigée vers le Soleil (on a remarqué des comètes se divisant et aussi des comètes à deux queues). Si $Q_2(d_1) < 0$ la comète peut ne pas avoir de queue ou bien si elle en a, elle est dirigée vers le Soleil.

Il serait encore plus naturel d'utiliser pour **F** la décomposition

$$\mathbf{F} = m_2\left[-f\,\frac{m_1+m_2}{r^2}+\frac{C^2}{r^3}-\frac{v^2}{r}+\frac{\dot r^2}{r}+\frac{(\dot\omega r+2\omega\dot r)\dot r}{C/r-\omega r}\right]\mathbf{u}- \tag{13}$$
$$-m_2\,\frac{\dot\omega r+2\omega\dot r}{C/r-\omega r}\,\mathbf{v},\quad \mathbf{v}=\dot r\mathbf{u}+r\dot\theta\mathbf{n},$$

de laquelle on peut déduire qu'on pourrait avoir plusieurs couronnes de répulsion. Il en résulterait un mécanisme capable d'expliquer l'apparition des anneaux dans une nébuleuse primaire sous la forme d'un disque en rotation, en particulier d'expliquer l'apparition des anneaux autour de certaines planètes. On pourrait expliquer, peut-être, les ceintures de radiation autour de la Terre (sourtout si on utilisait aussi les résultats fournis par l'étude qualitative des trajectoires qui résultent de (5)).

La décomposition (13) a presque la même forme que dans le cas de la théorie de la relativité (v. [2], p. 101).

La concordance de tout ceci avec les observations pourra être vérifiée au moment où les constantes qui interviennent dans le cadre de cette théorie seront précisées.

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